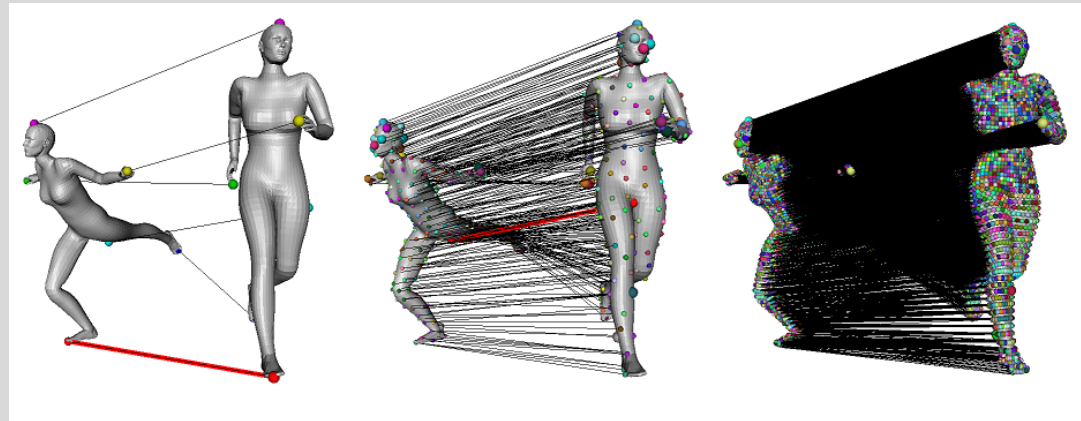
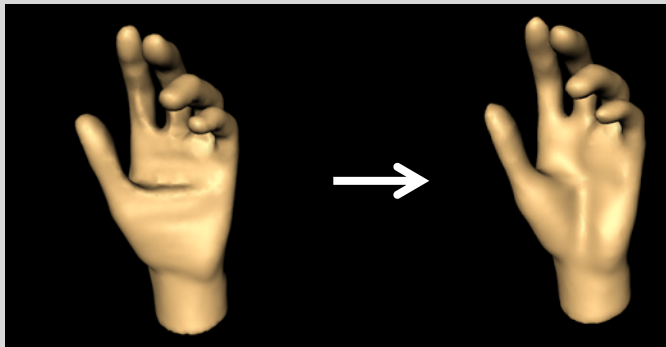
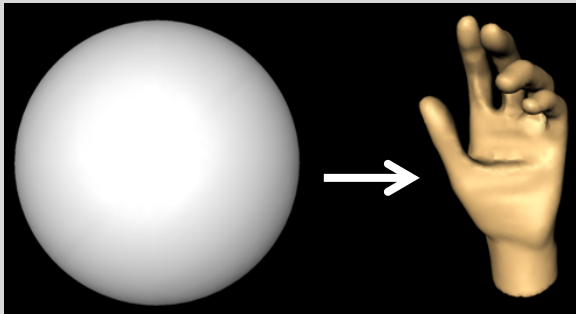


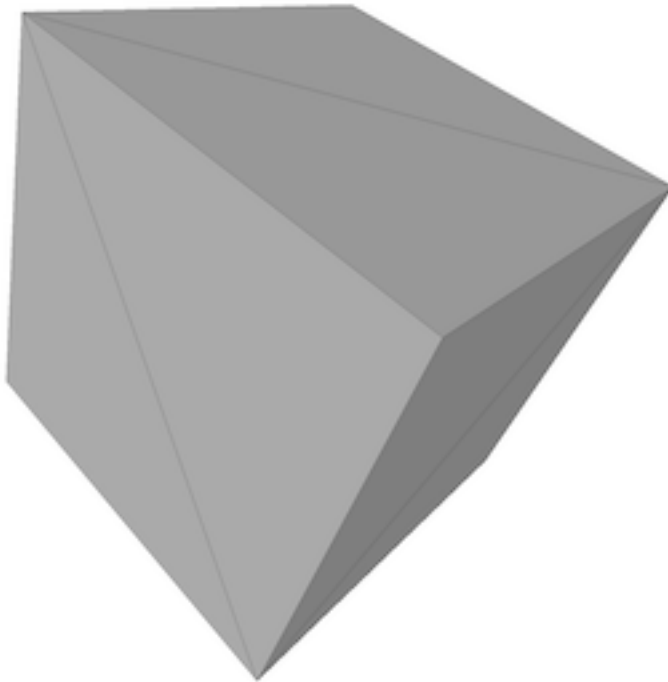
Mesh Processing: From Creation to Comparison



Assoc. Prof. Yusuf Sahilliođlu

Surface Mesh

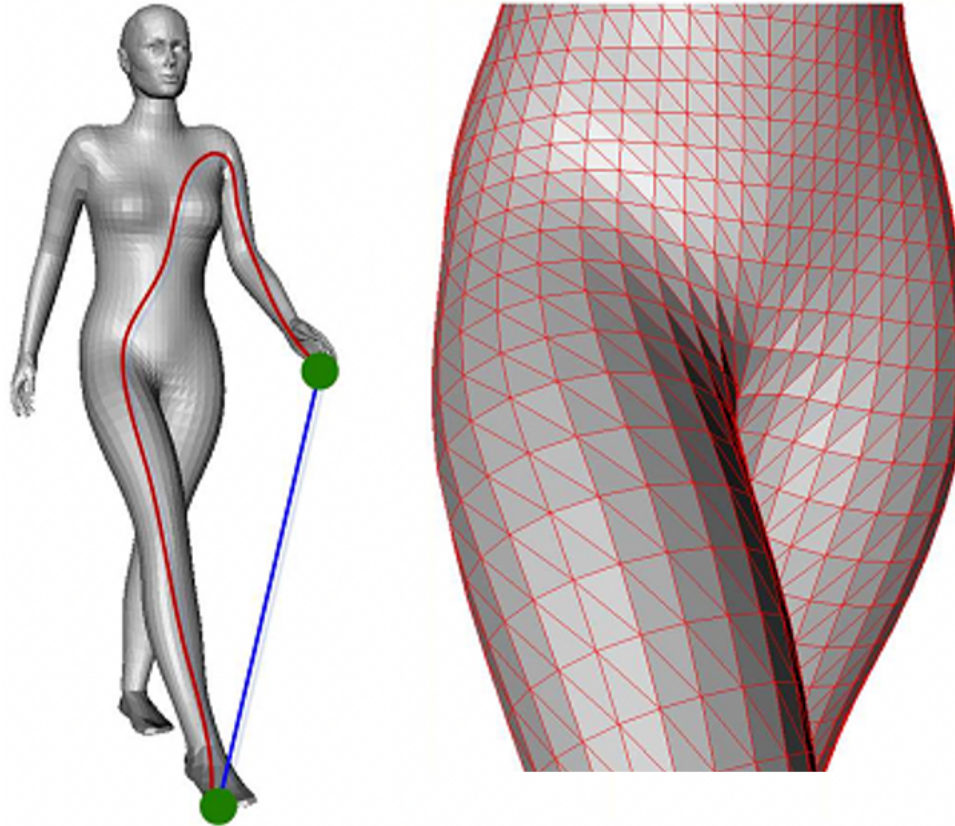
Representing geometry and topology as a polygon mesh.



```
1 v 1.000000 -1.000000 -1.000000
2 v 1.000000 -1.000000 1.000000
3 v -1.000000 -1.000000 1.000000
4 v -1.000000 -1.000000 -1.000000
5 v 1.000000 1.000000 -0.999999
6 v 0.999999 1.000000 1.000001
7 v -1.000000 1.000000 1.000000
8 v -1.000000 1.000000 -1.000000
9 f 2 3 4
10 f 8 7 6
11 f 5 6 2
12 f 6 7 3
13 f 3 7 8
14 f 1 4 8
15 f 1 2 4
16 f 5 8 6
17 f 1 5 2
18 f 2 6 3
19 f 4 3 8
20 f 5 1 8
```

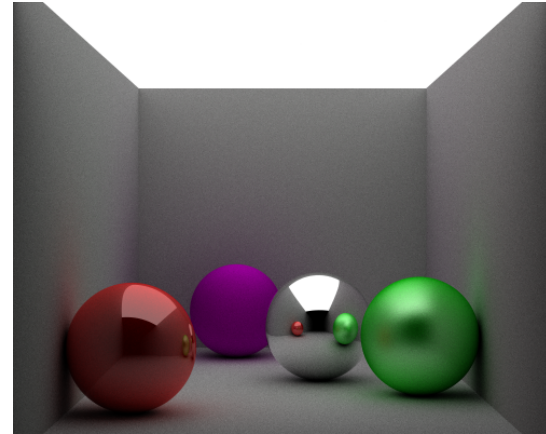
Surface Mesh

Representing geometry and topology as a polygon mesh.

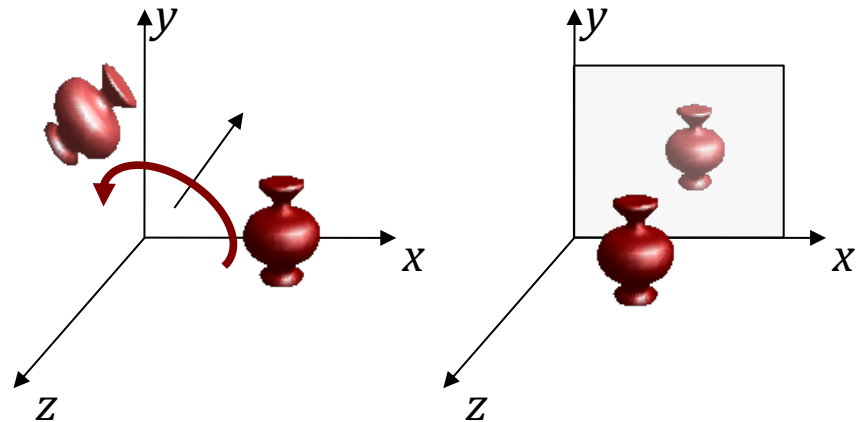


MP vs. CG

- Computer graphics //CENG 477
 - Rendering
 - Ray tracing
 - Rasterization

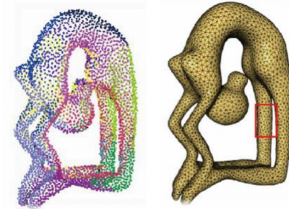


- Transformations



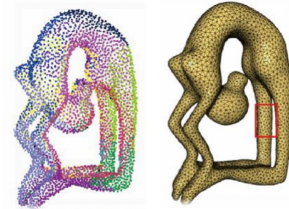
MP vs. CG

- Mesh Processing //CENG 789
 - Reconstruction

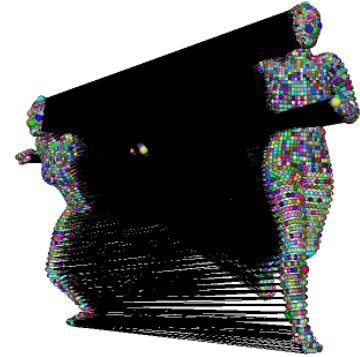
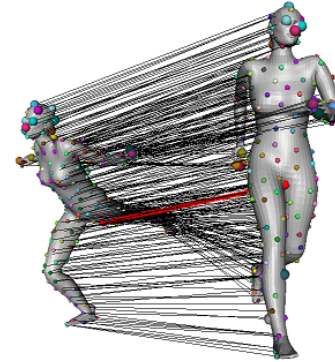
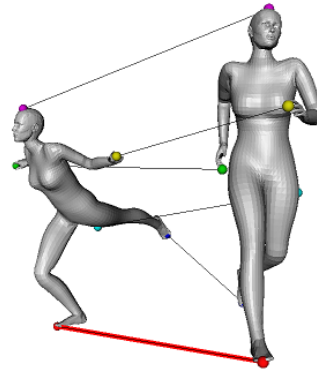


MP vs. CG

- Mesh Processing //CENG 789
 - Reconstruction

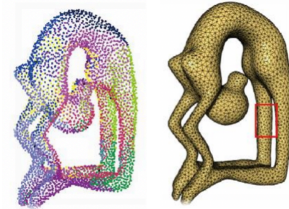


- Analysis

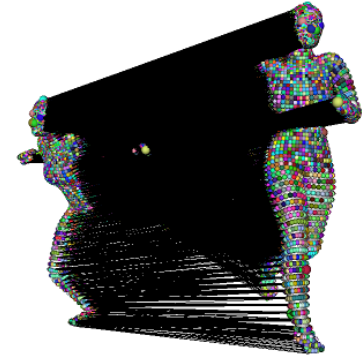
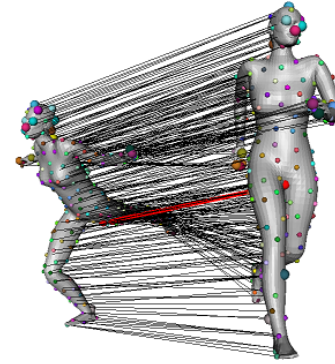
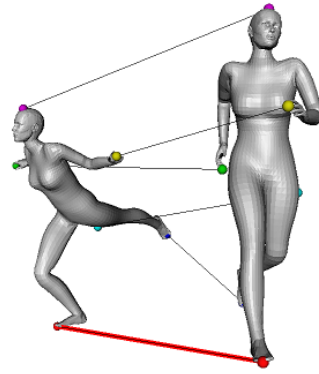


MP vs. CG

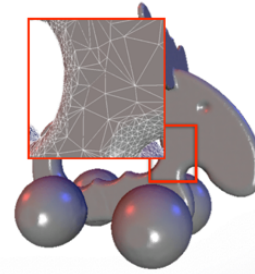
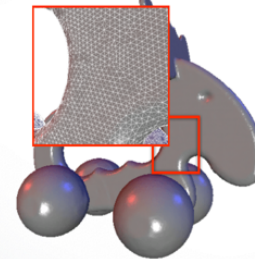
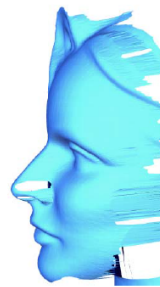
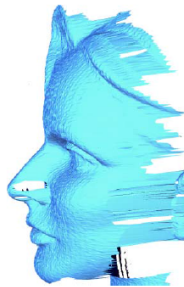
- Mesh Processing //CENG 789
 - Reconstruction



- Analysis



- Smoothing/Remeshing



MP vs. CG

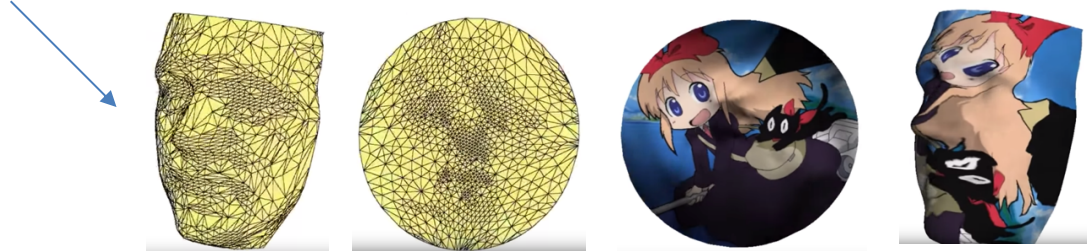
- Mesh Processing //CENG 789
 - Parameterization
 - Deformation
 - Registration
 - Fabrication



- I'll focus on my papers on Reconstruction and Analysis
(Creation) (Comparison)

MP vs. CG

- Mesh Processing //CENG 789
 - Parameterization
 - Deformation
 - Registration
 - Fabrication



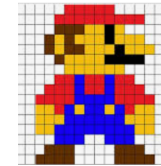
- I'll focus on my papers on Reconstruction and Analysis
(Creation) (Comparison)

MP vs. IP

- Image Processing //CENG 466

- Regularity

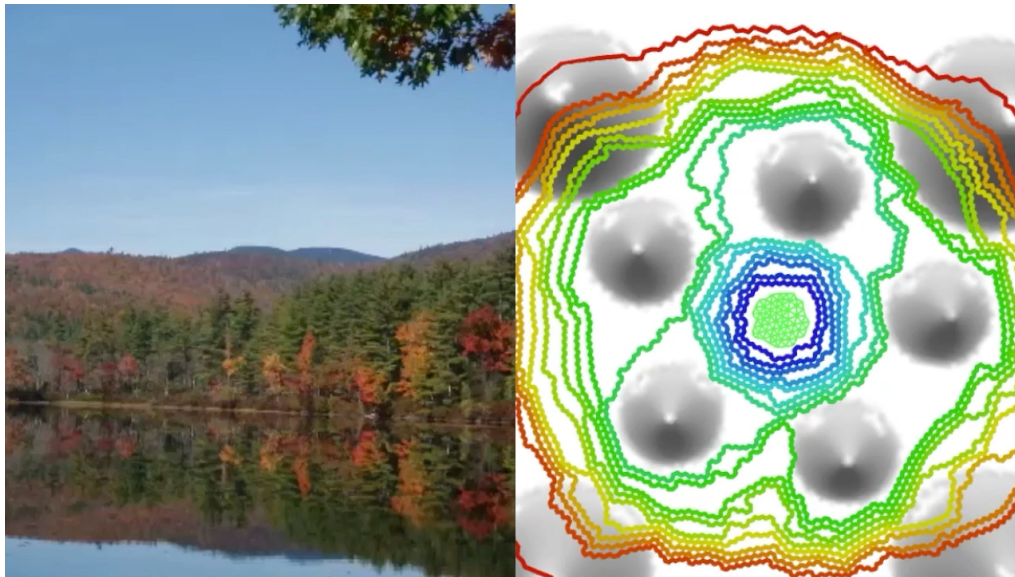
- IP: Every pixel has 4 neighbors



- MP: Every edge is incident to 2 faces

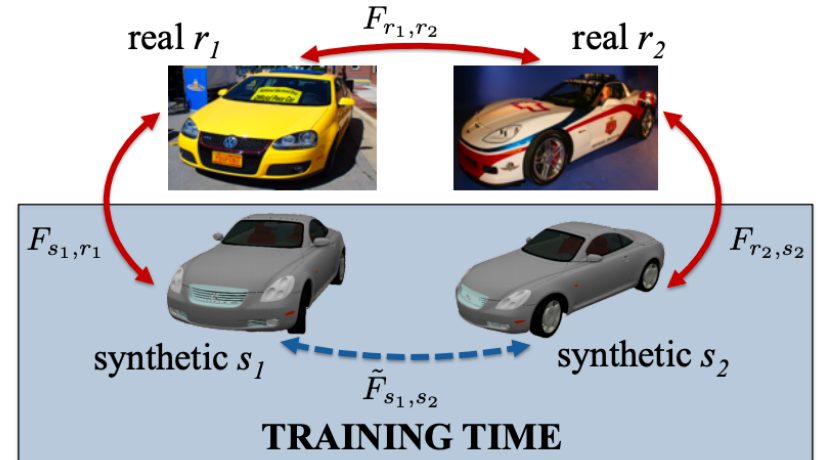
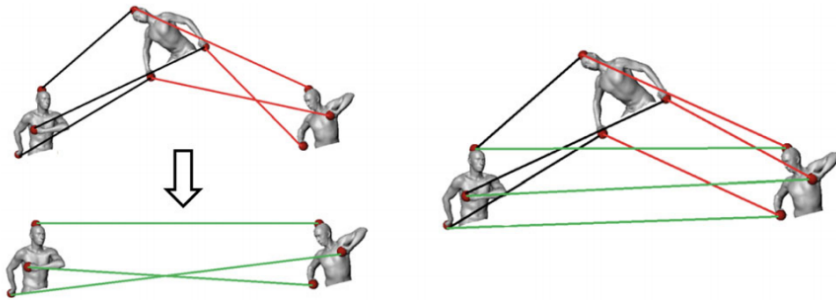


- Exchange of ideas IP \rightarrow MP



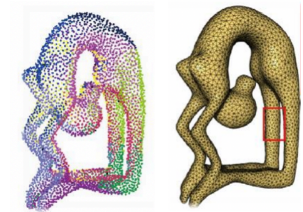
MP vs. IP

- Image Processing //CENG 466
 - Regularity
 - IP: Every pixel has 4 neighbors
 - MP: Every edge is incident to 2 faces
- Exchange of ideas MP \rightarrow IP



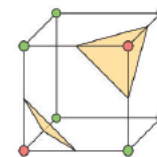
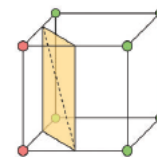
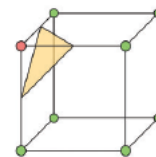
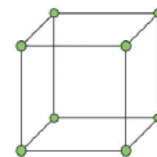
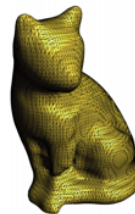
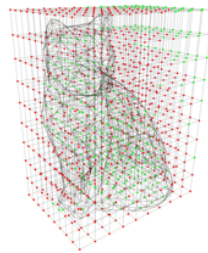
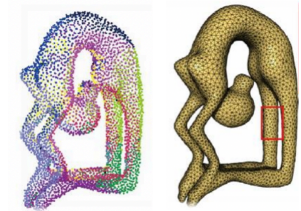
Reconstruction

- Input: 3D point samples
- Output: Surface fit to the point samples
- 3D point samples can be acquired **passively** or **actively**
 - **Stereoscopic images**
 - **Multiple silhouettes**
 - **Emitters**
 - **LIDAR, Laser Scanner, Kinect, ToF**



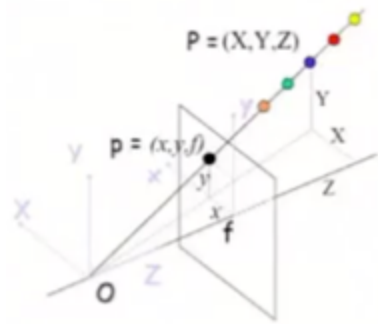
Reconstruction

- Input: 3D point samples
- Output: Surface fit to the point samples
- 3D point samples can be acquired **passively** or **actively**
 - Stereoscopic images
 - Multiple silhouettes
 - Emitters
 - LIDAR, Laser Scanner, Kinect, ToF
- Alternative input: Scalar field defined over a 3D grid (CT)
 - Defines surface implicitly
 - Implicit methods, e.g., Marching Cubes, to extract the surface.



Reconstruction

- Input: 3D point samples; how to get them?
- Stereoscopy

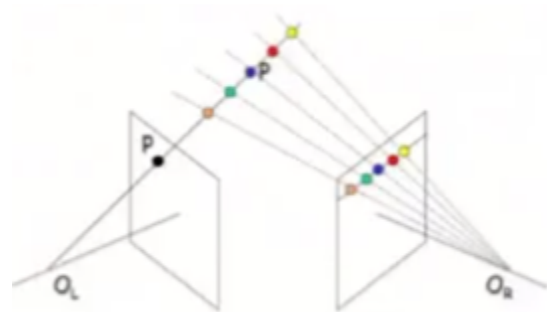


$$x = \frac{fX}{Z} = \frac{fkX}{kZ}$$

$$y = \frac{fY}{Z} = \frac{fkY}{kZ}$$

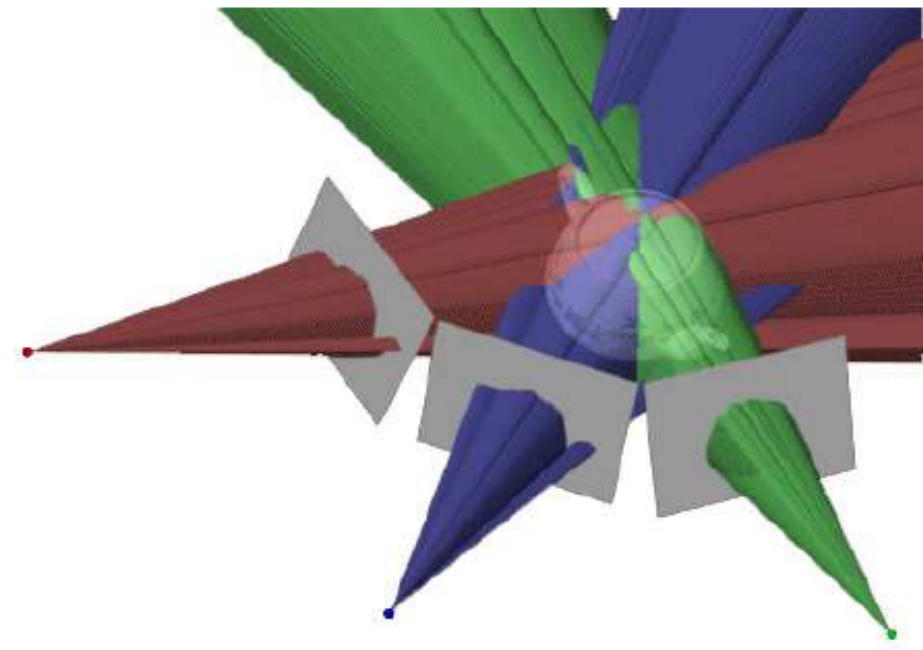
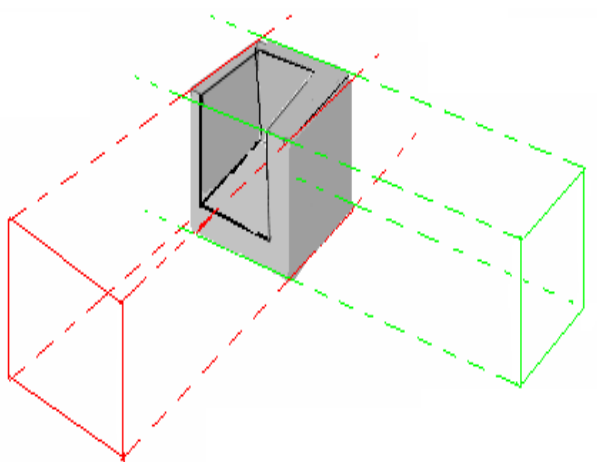
For any $k \neq 0$

Depth ambiguity handled by a second image



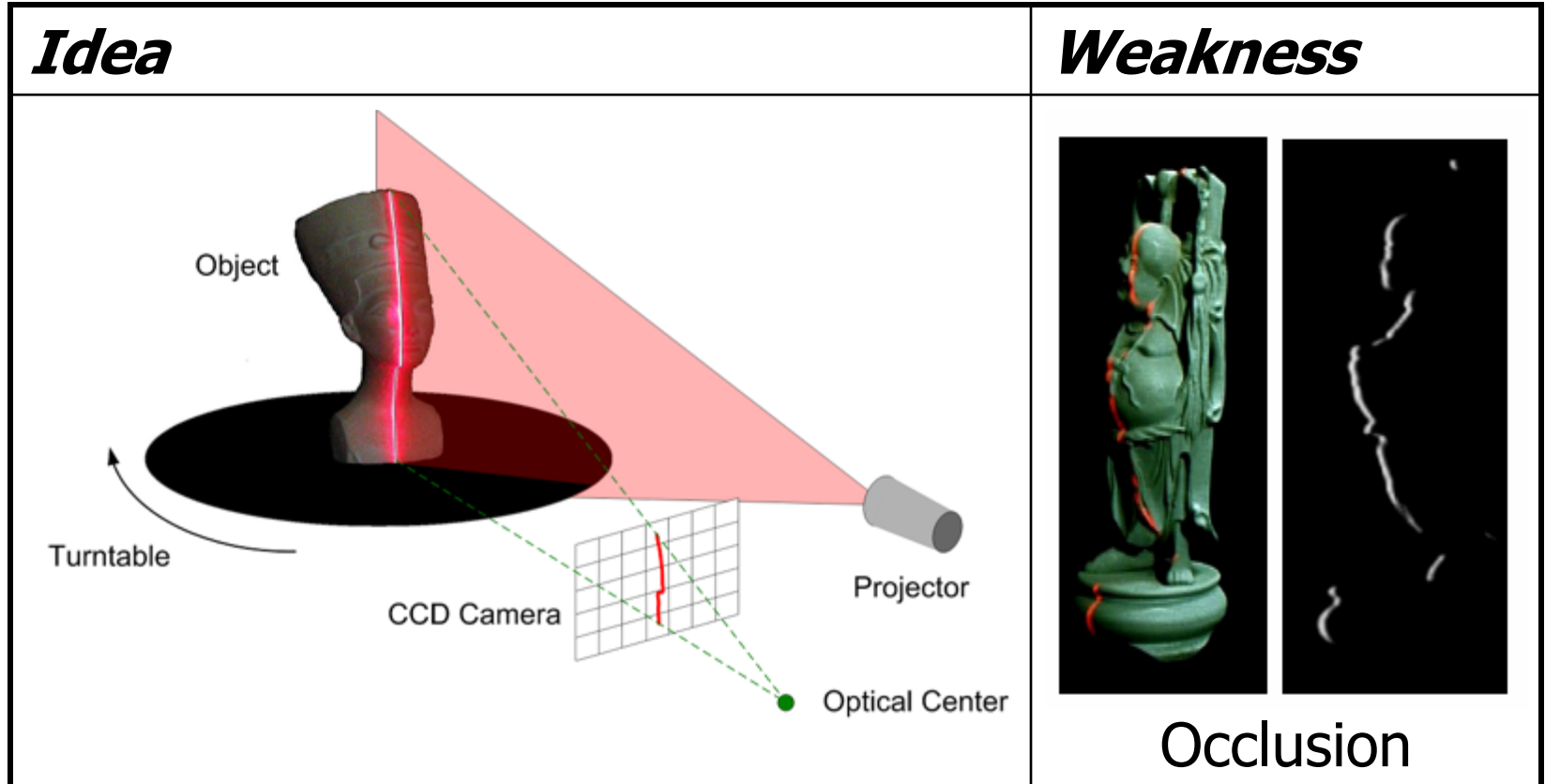
Reconstruction

- Input: 3D point samples; how to get them?
- Silhouettes

<i>Idea</i>	<i>Weakness</i>
	 <p data-bbox="1255 1185 1738 1242">Hidden Concavity</p>

Reconstruction

- Input: 3D point samples; how to get them?
- Structured light



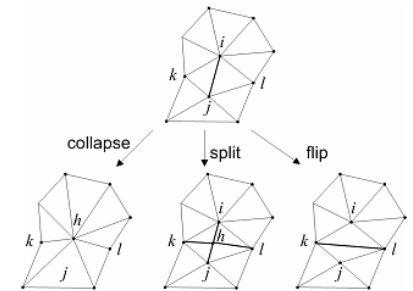
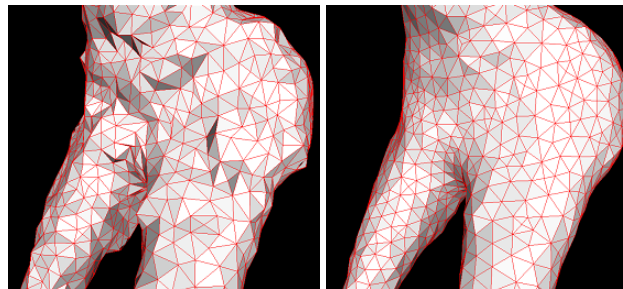
Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?

Iterate

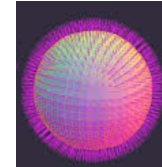
- Move each vertex P with $v(P, B)$ in the direction of its normal $N(P)$, as F_{ext} suggests: $\mathbf{F}_{\text{ext}}(P, B) = v(P, B) \cdot \mathbf{N}(P)$
- Regularize the mesh by F_{int}
- Collapse edges with length smaller than ϵ_{min}
- Split edges with length exceeding $\epsilon_{\text{max}} = 2\epsilon_{\text{min}}$
- Flip edges where necessary, favoring the vertices with valences close to 6

Till convergence

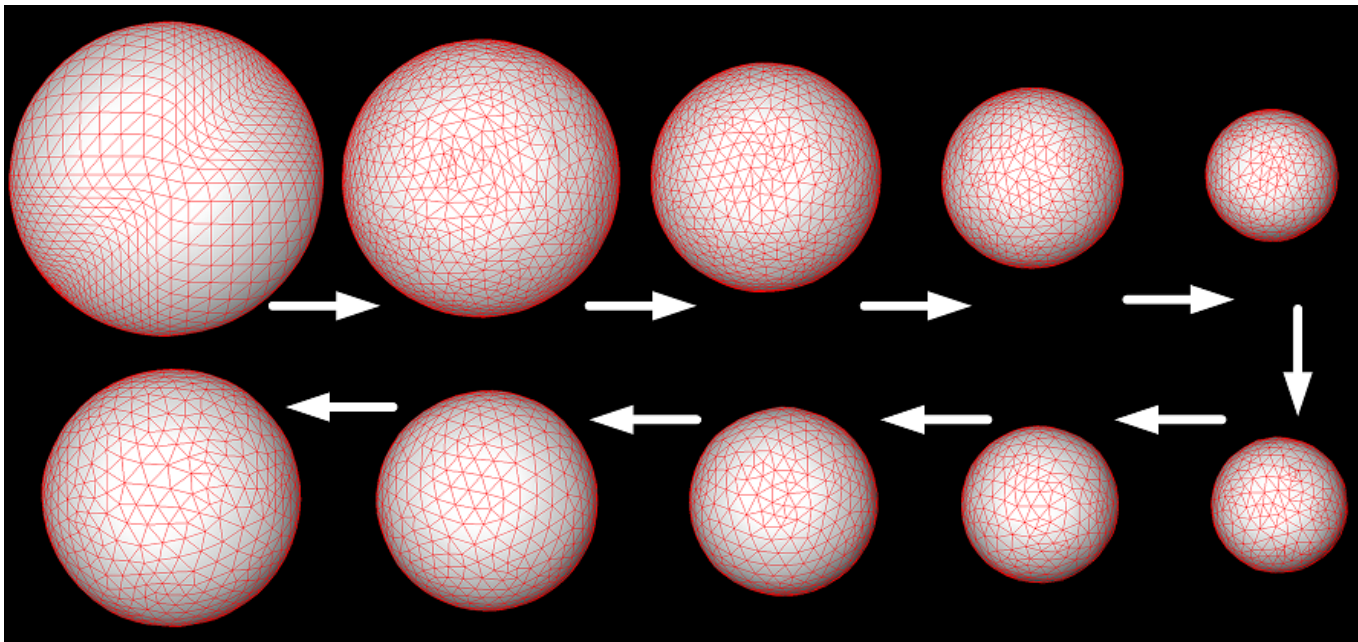


Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?



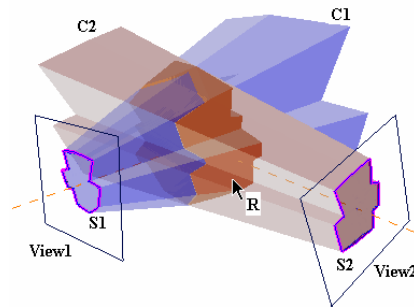
F_{ext}: constant external force ($F(P) = -\epsilon_{min}/2 * N(P)$)



Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?

F_{ext} : force based on silhouettes



$$\mathbf{F}_{\text{ext}}(P, B) = v(P, B) \cdot \mathbf{N}(P)$$

$$v(P) = \varepsilon_{\min} f(P) = \varepsilon_{\min} \min_n \left\{ G \left[\text{Proj}_{I_n} (P) \right] - 0.5 \right\}$$

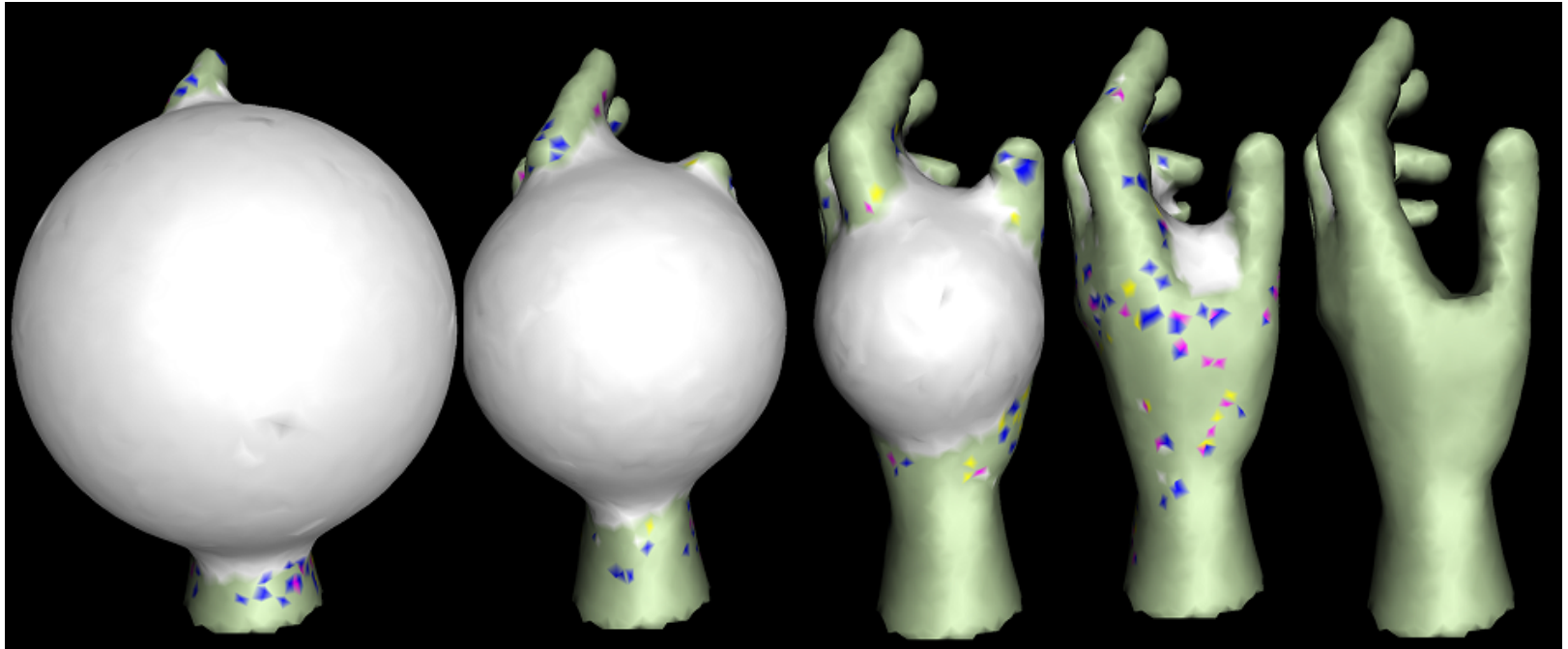


$$G(x', y') = (1 - \alpha)((1 - \beta)I(\lfloor x' \rfloor, \lfloor y' \rfloor) + \beta I(\lfloor x' \rfloor, \lfloor y' \rfloor + 1)) \\ + \alpha((1 - \beta)I(\lfloor x' \rfloor + 1, \lfloor y' \rfloor) + \beta I(\lfloor x' \rfloor + 1, \lfloor y' \rfloor + 1))$$

Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?

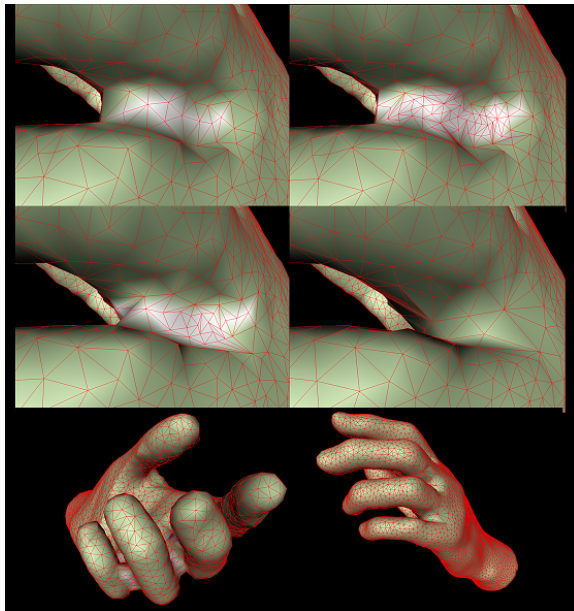
Fext: force based on silhouettes



Reconstruction

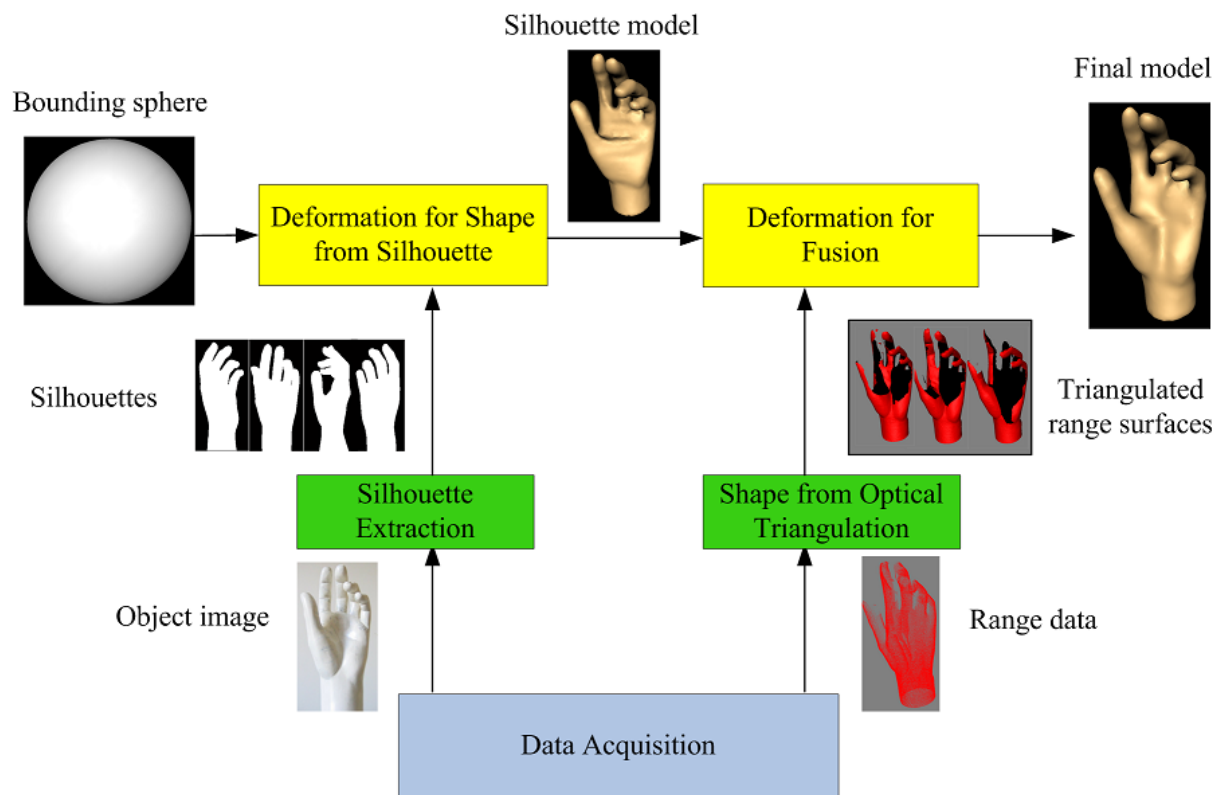
- Input: 3D point samples
- Output: Surface; how to get it?

Fext: force based on silhouettes



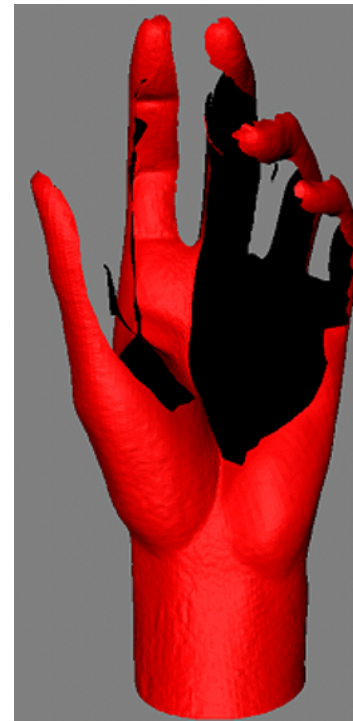
Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?
- Hidden concavity problem solved using range/laser surface



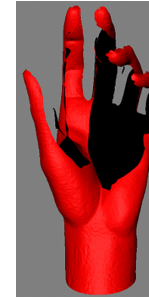
Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?
- Hidden concavity problem solved using range/laser surface

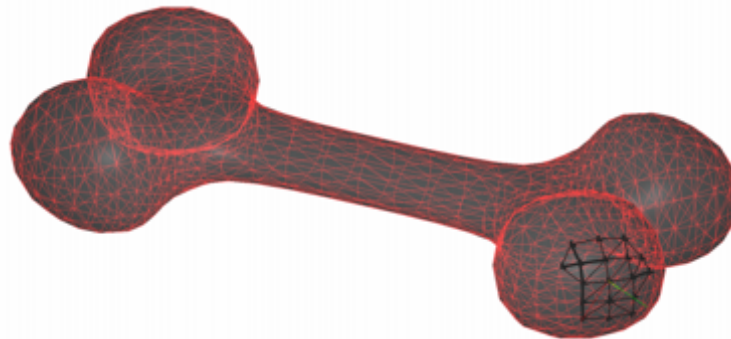


Reconstruction

- Input: 3D point samples
- Output: Surface; how to get it?
- Mesh up laser point cloud as follows

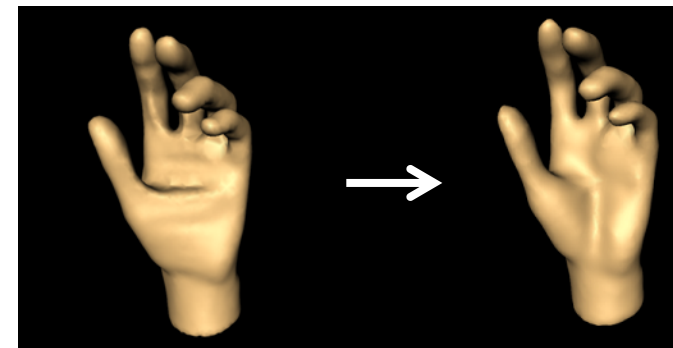
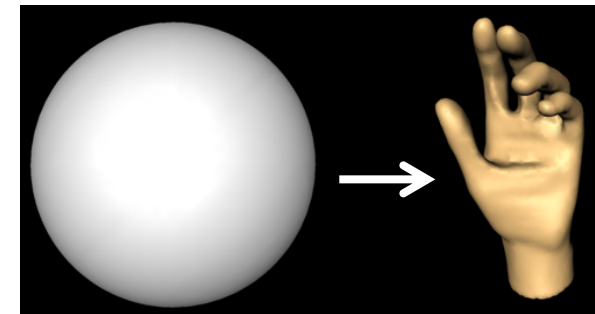
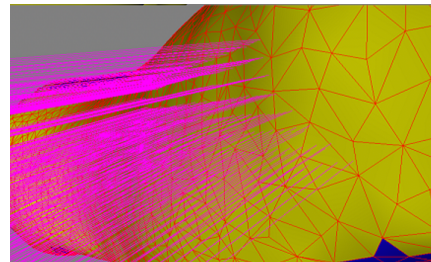
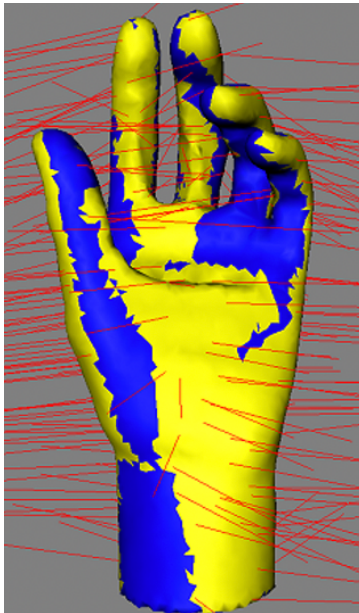


- Find local neighborhood L_i of each point in the 3D point cloud input
- For each L_i compute tangent plane using PCA
- Project all points in L_i to the tangent plane and compute their 2D Delaunay triangulation
- Merge all these local triangulations into a global one



Reconstruction

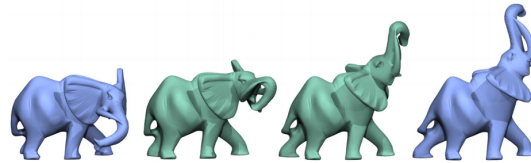
- Input: 3D point samples
- Output: Surface; how to get it?
- Refine the silhouette-based mesh using an updated $F_{ext}(P)$ based on carvers assigned to triangles that share the vertex P



Comparison/Correspondence

- Once we have the meshes (reconstruction), we want to relate them with each other to enable nice apps, such as

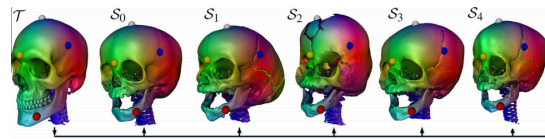
- Shape interpolation:



- Deformation transfer:



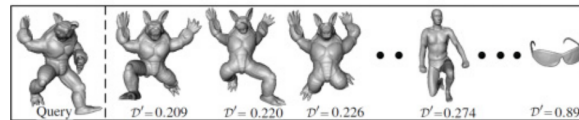
- Attribute transfer:



- Shape registration:



- Shape matching:



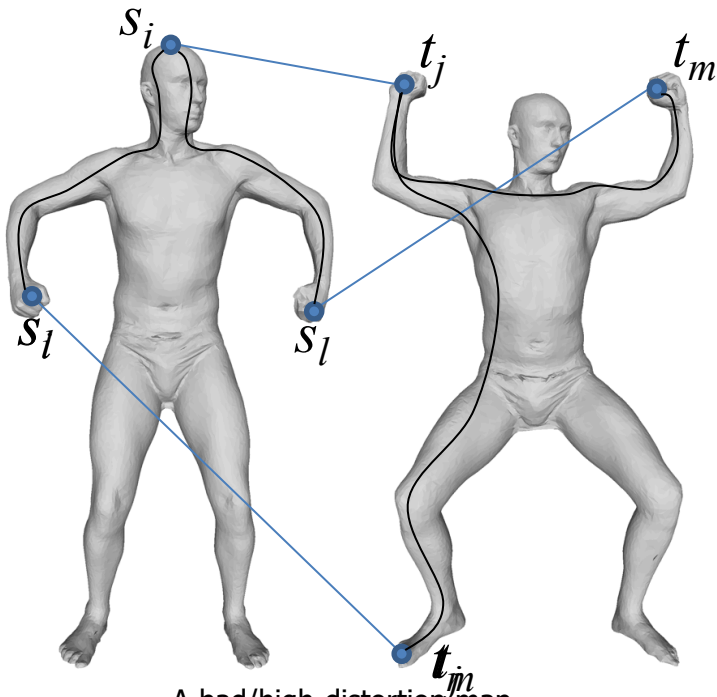
- Statistical analysis:



Correspondence

- Solution idea
 - Quantify the quality of a given map
 - Then search the map space using this metric

$$\mathcal{D}_{\text{iso}}(\phi) = \frac{1}{|\phi|} \sum_{(s_i, t_j) \in \phi} \left(\frac{1}{|\phi'|} \sum_{(s_l, t_m) \in \phi'} |d_g(s_i, s_l) - d_g(t_j, t_m)| \right)$$

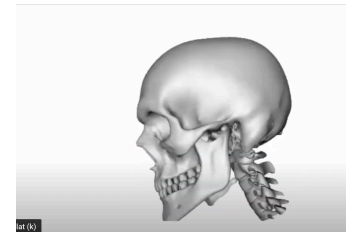
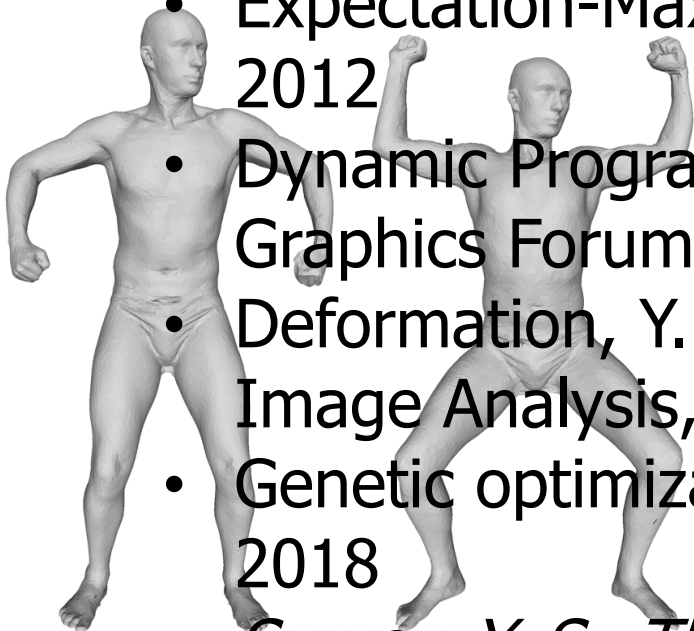


A bad/high-distortion map.

$$|.34 - .98| = .64 \text{ ☹}$$

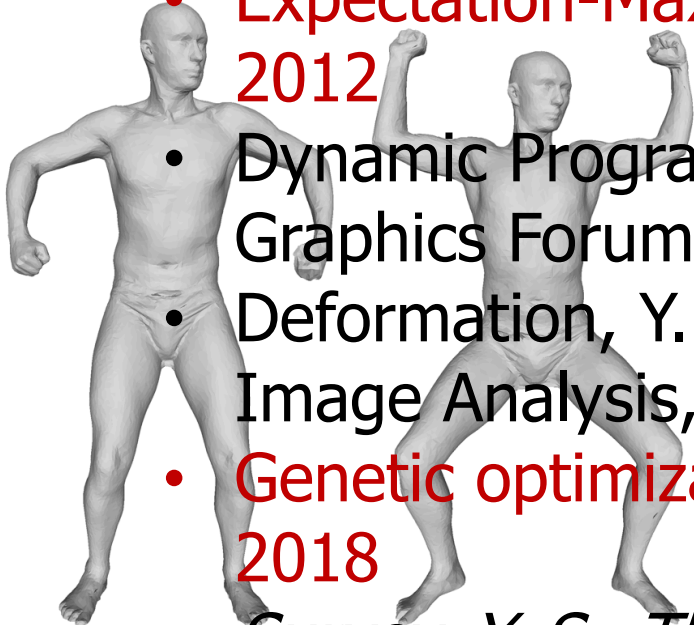
Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
 - Greedy optimization, Y. S., Y. Yemez, CVPR, 2010
 - Combinatorial optimization, Y. S., Y. Yemez, Computer Graphics Forum, 2011, 2012, 2013, 2014
 - Expectation-Maximization (EM), Y. S., Y. Yemez, PAMI, 2012
 - Dynamic Programming, Y. S., Y. Yemez, Computer Graphics Forum, 2014
 - Deformation, Y. S., L. Kavan, Medical Image Analysis, 2015
 - Genetic optimization, Y. S., Transactions on Graphics 2018
 - *Survey, Y. S., The Visual Computer, 2020*



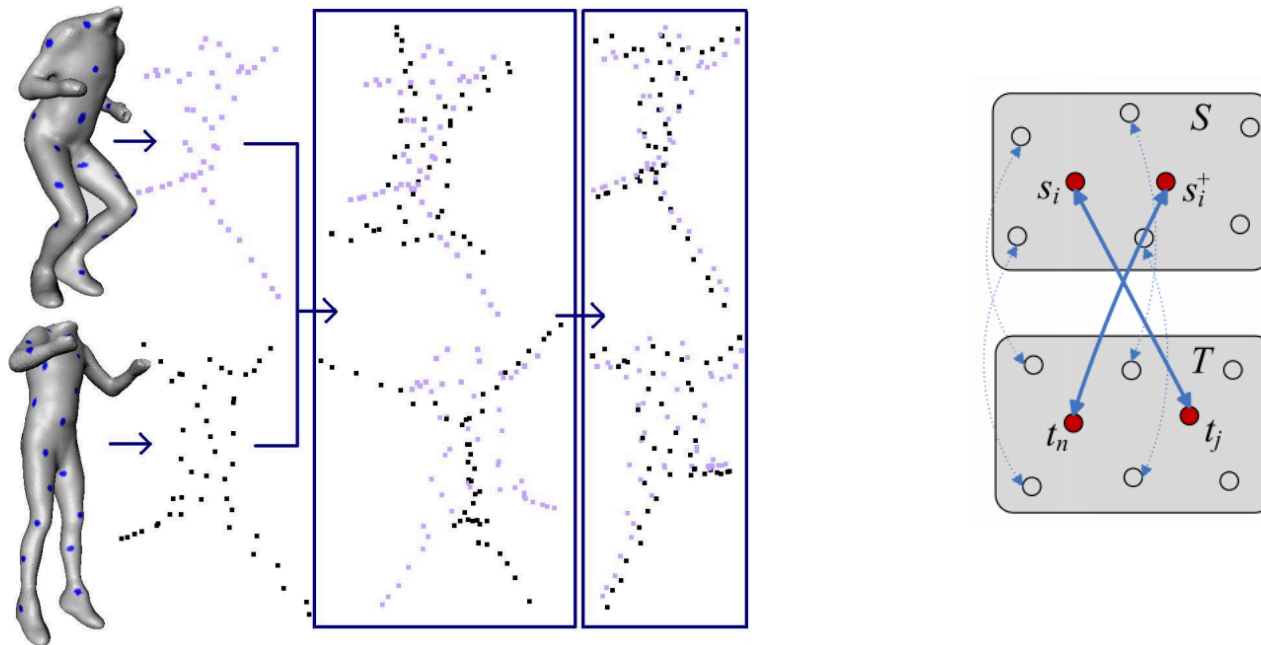
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 - Genetic optimization, Y. S., Transactions on Graphics 2018
 - Survey, Y. S., *The Visual Computer*, 2020



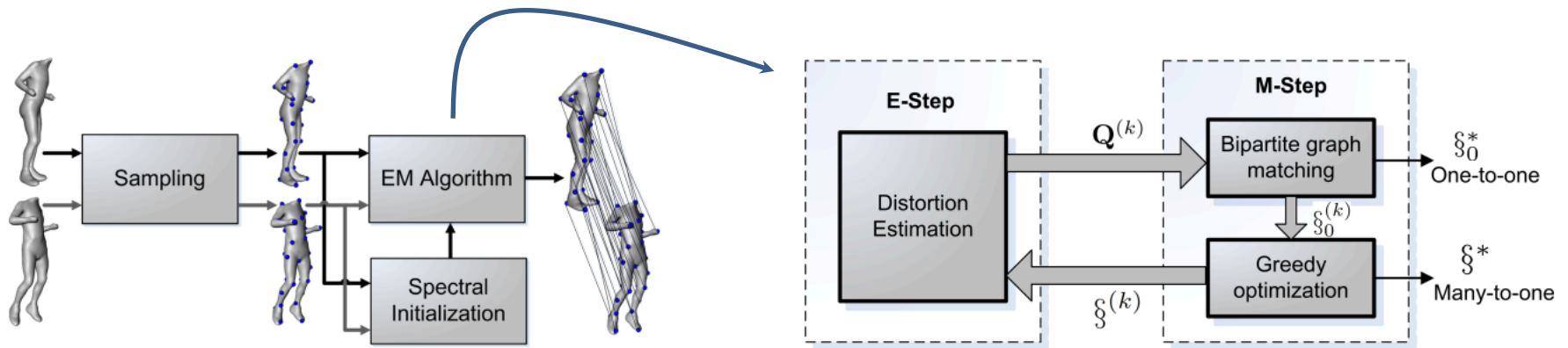
Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
- Initial correspondence via MDS (left) is refined by greedy optimization based on neighbor voting (right).



Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
- Isometric cost of matching s_i to t_j for all pairs (Q matrix in E-Step) guides graph matching and refinement which results in a better map to estimate Q (M-step). Repeat.



Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
- Represent a permutation as a chromosome and evolve many of them into the fittest one that yields the min-distortion map

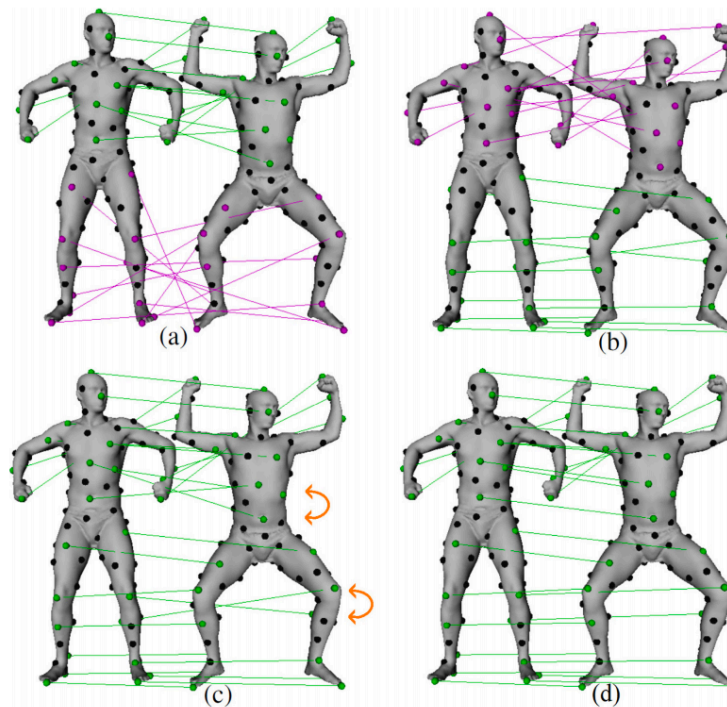
1 20 3 .. 44 45 .. N
81 7 4 .. N 59 .. 22
77 6 9 .. 26 10 .. 5
:
24 7 9 .. 33 48 .. 8

Genetic Algorithm

1 20 3 .. 44 45 .. 99

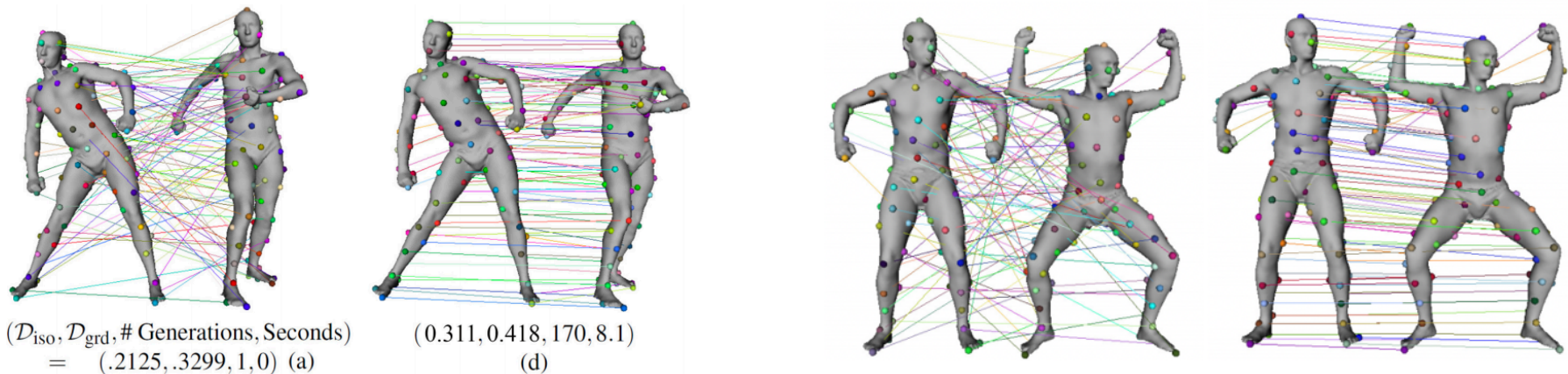
Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
- Represent a permutation as a chromosome and evolve through genetic operators crossover and mutation



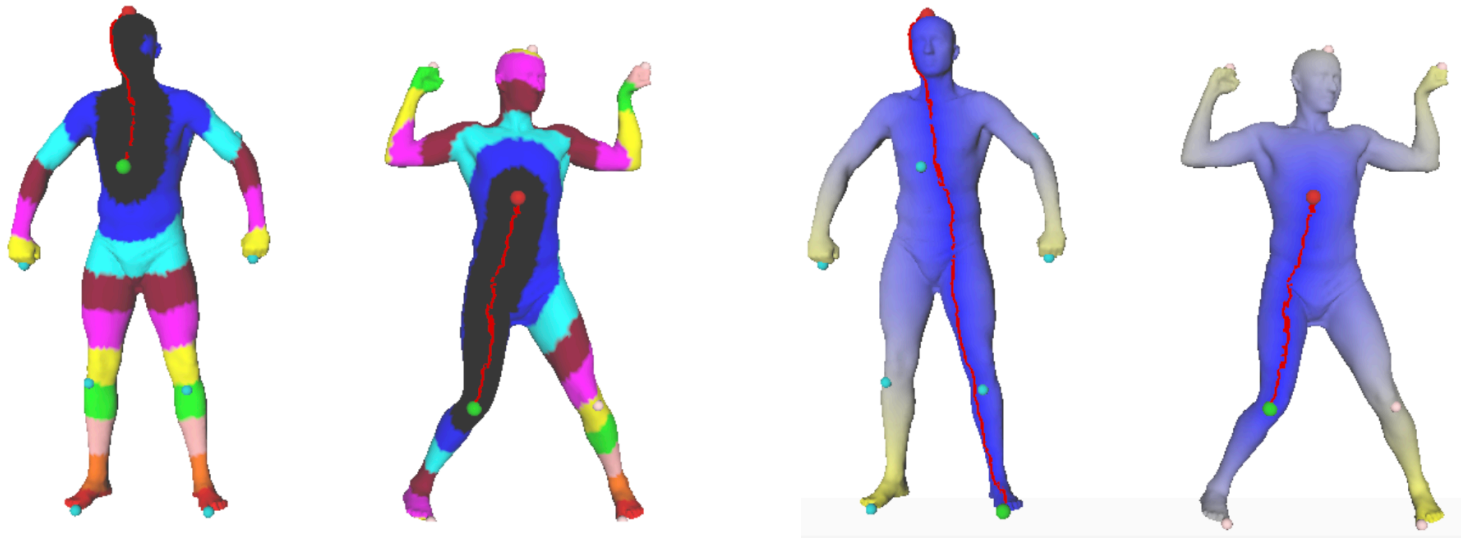
Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
- Represent a permutation as a chromosome and evolve through genetic operators crossover and mutation



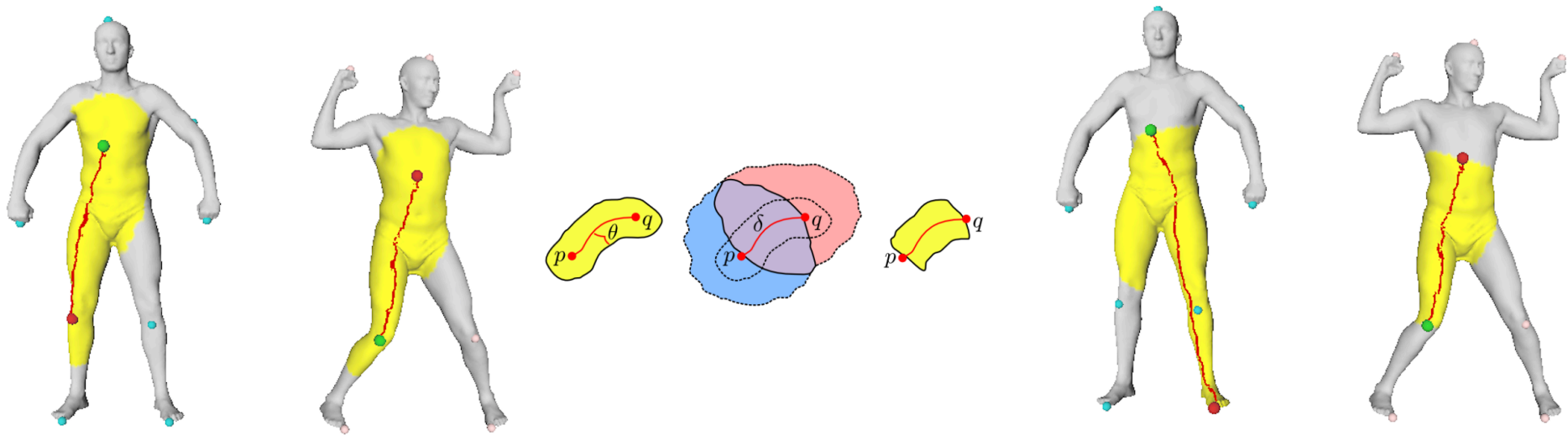
Correspondence

- Looking at all $N!$ permutations is infeasible
- Minimize this metric (or its variants) using:
- Improved bilateral maps: scale-invariance, fuzzy voting.



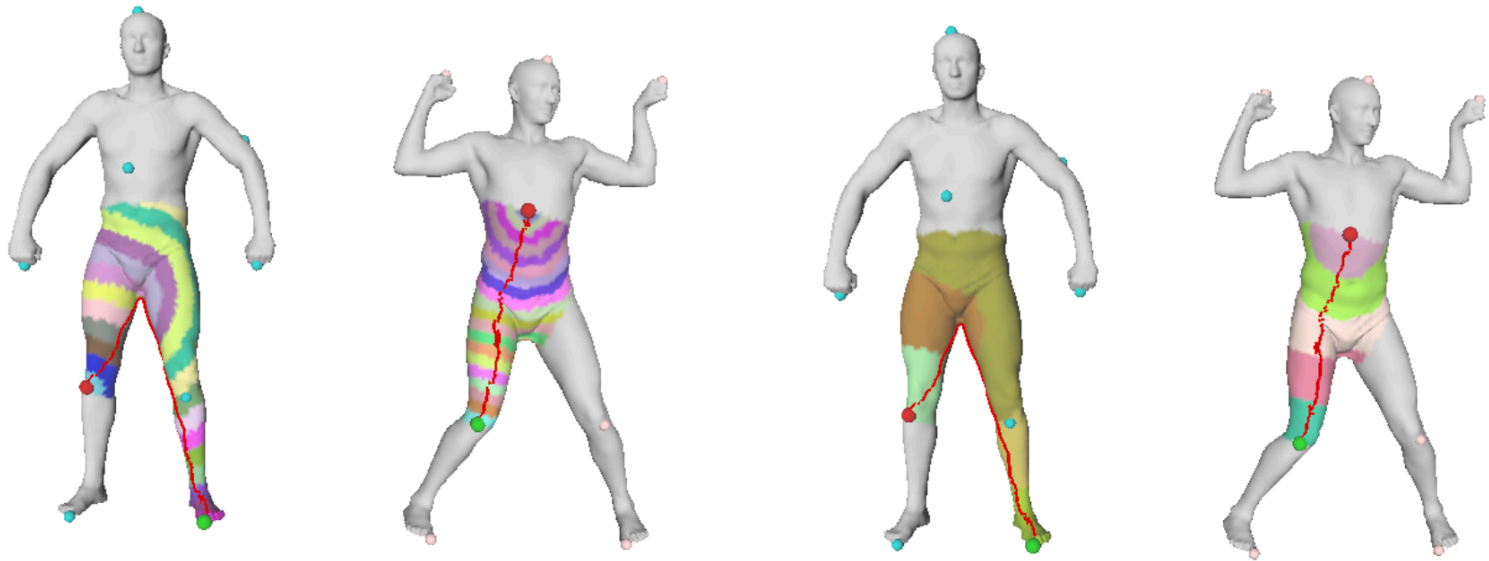
Correspondence

- Looking at all $N!$ permutations is infeasible
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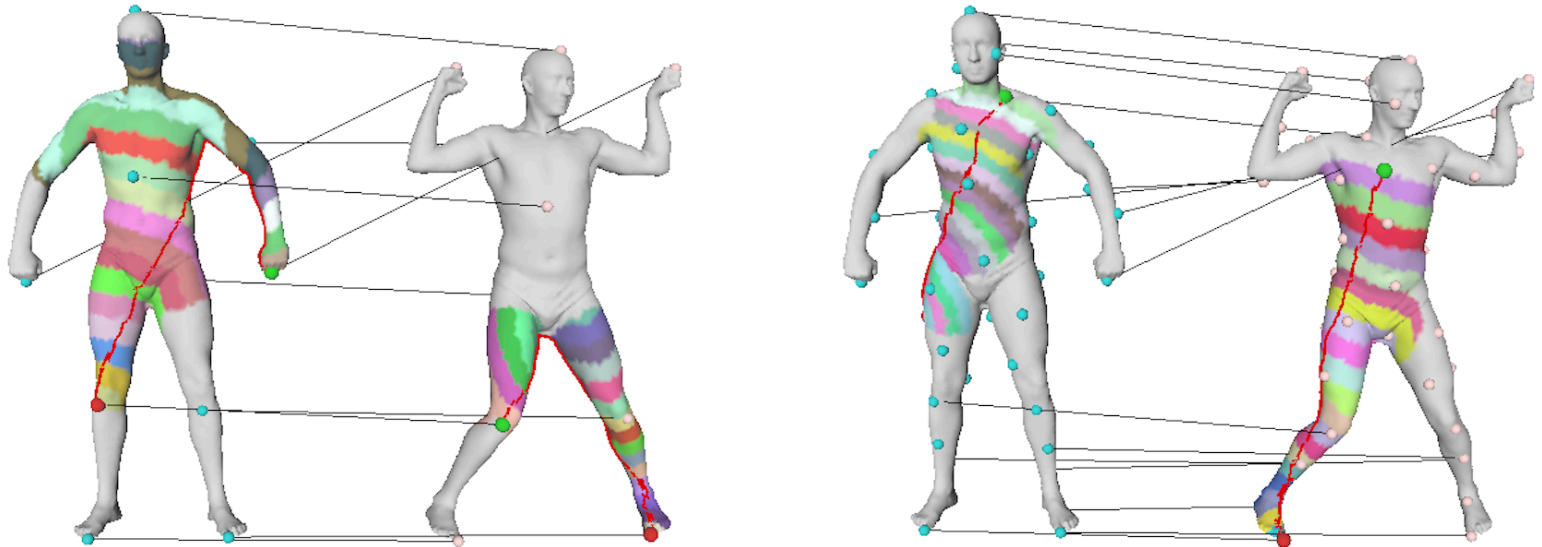
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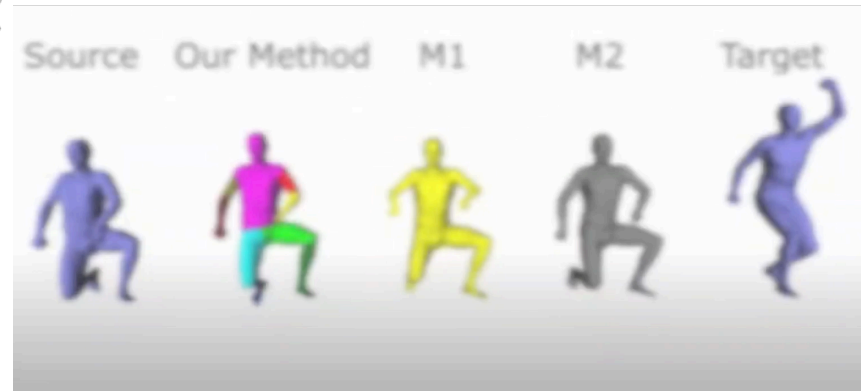
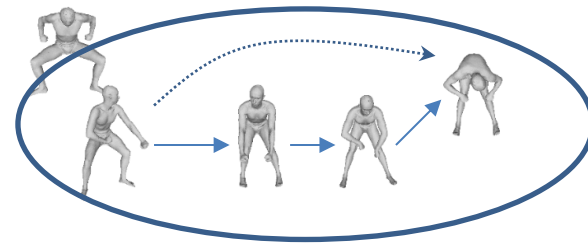
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Interpolation

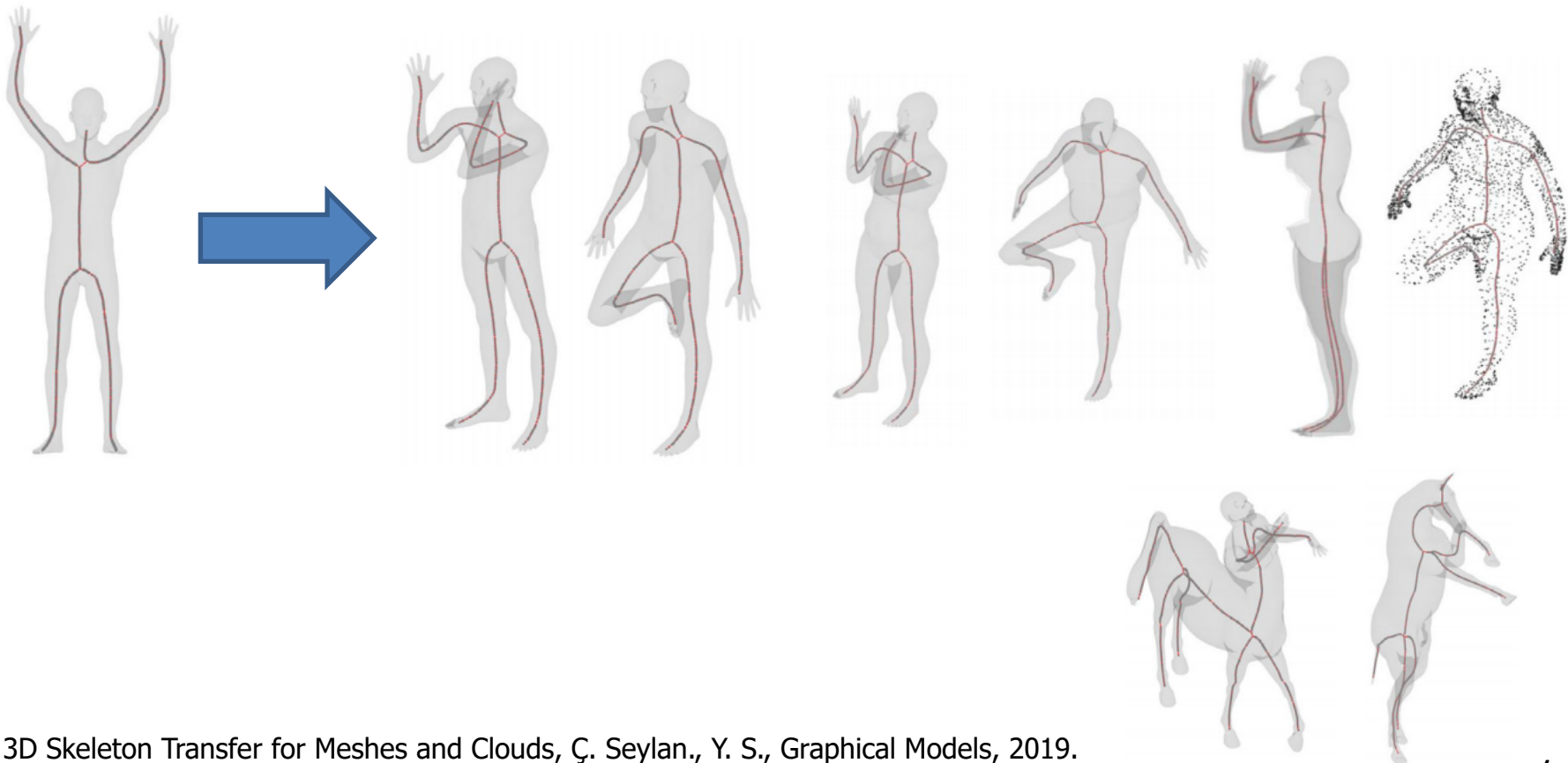
- Correspondence in action: shape interpolation
- Interpolate through the shortest path of inter-shapes



$$v_{\text{result}}^k(\alpha) = \sum_{i=0}^m v_i^k b_{i,m}(\alpha), \quad b_{i,m}(\alpha) = \binom{m}{i} \alpha^i (1-\alpha)^{m-i}$$

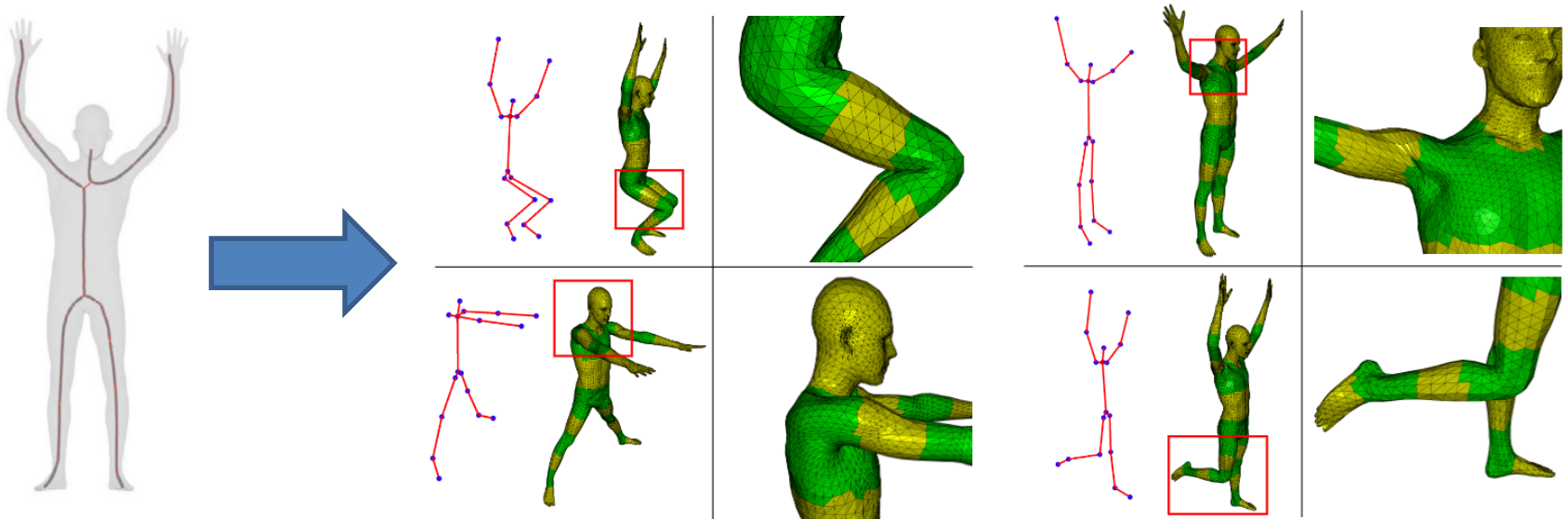
Skeleton

- Correspondence in action: skeleton extraction/transfer
- Transfer the skeleton in source mesh to the target mesh using surface mesh correspondences



Skeleton

- Correspondence in action: inverse problem, skin extract
- Transfer the source mesh to the target skeleton using skeleton correspondences



Thanks

Papers, codes, executables, lectures, ..: <http://ceng.metu.edu.tr/~ys>



Y. S., Assoc. Prof.

