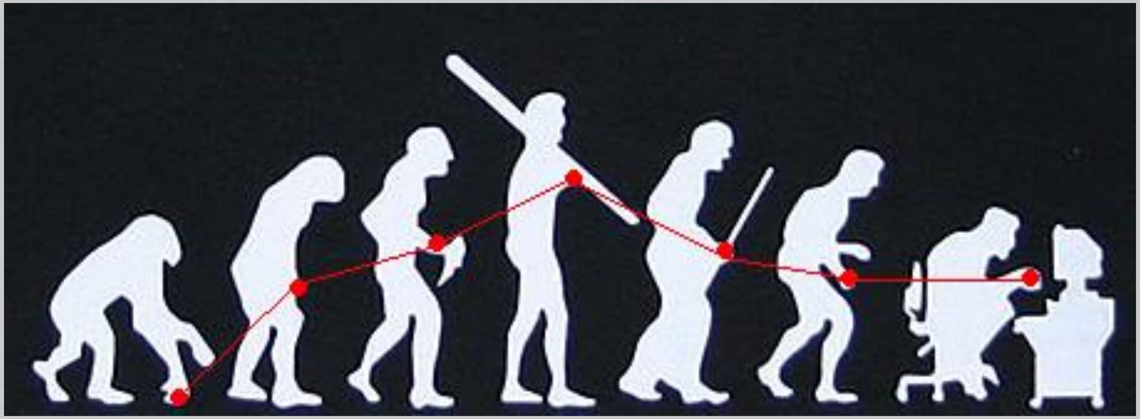
Algorithms for
3D Isometric Shape Correspondence



Yusuf Sahillioğlu
Computer Eng. Dept., Koç University, Istanbul, Turkey (PhD)
Computer Eng. Dept., METU, Ankara, Turkey (Asst. Prof.)

ODTÜ
METU

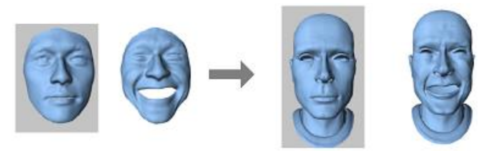
Problem Definition & Apps

Goal: Find a mapping between two shapes.

$$\boxed{\S : S \rightarrow T}$$

- ✓ Shape interpolation, animation.

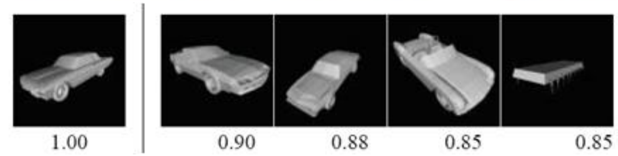
- ✓ Attribute transfer.



- ✓ Shape registration.

- ✓ Time-varying reconstruction.

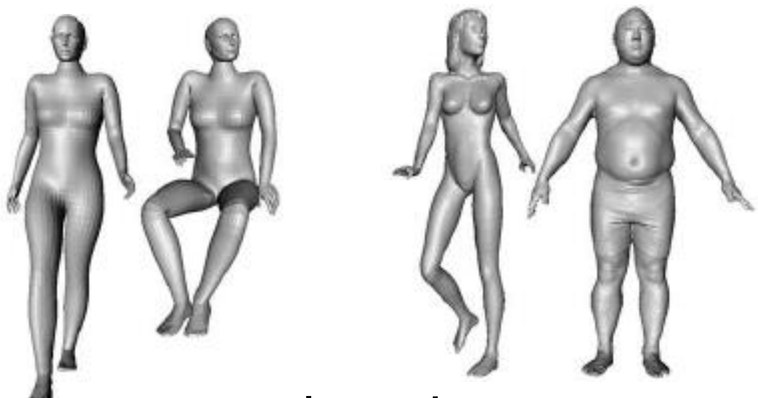
- ✓ Shape matching.

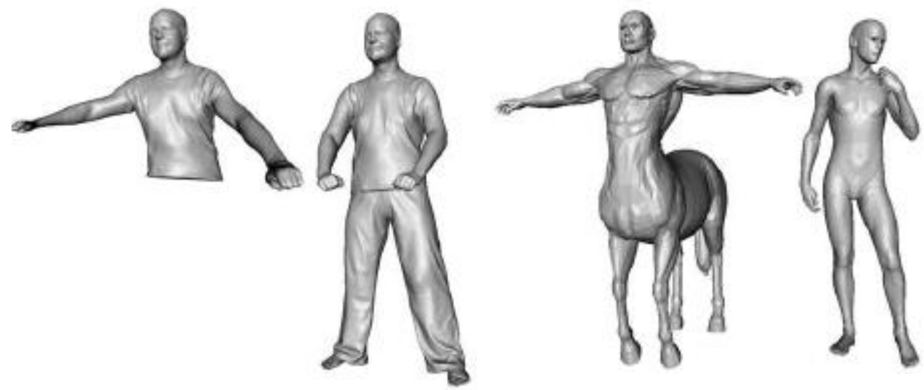


- ✓ Statistical shape analysis.

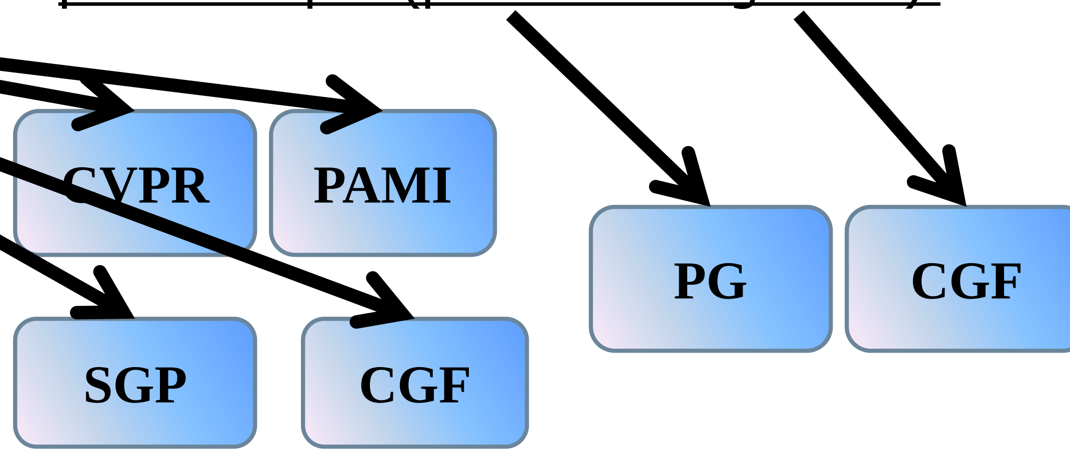
Scope

- ✓ Correspondence algorithms for (nearly) isometric *

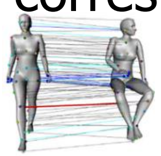




complete shapes. partial shapes (part or most general).



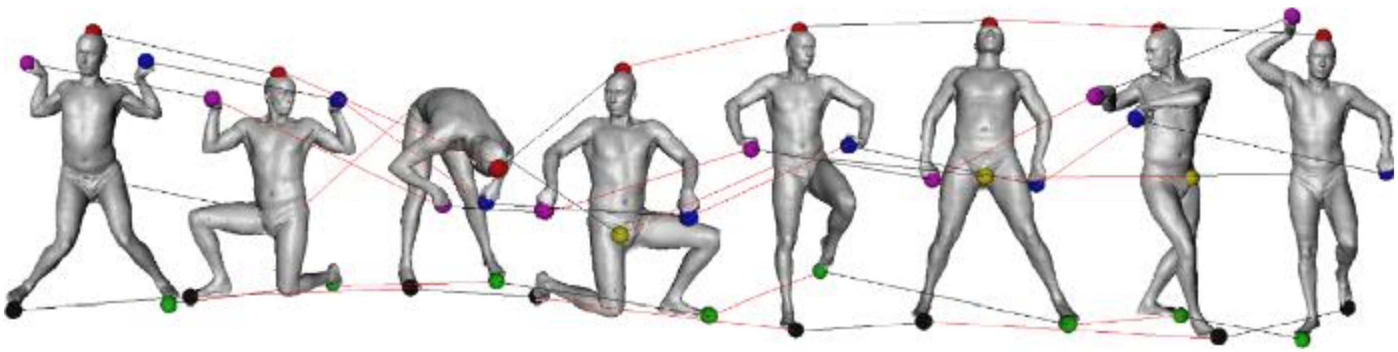
- ✓ Coarse correspondences.



- ✓ Dense correspondences.

Scope

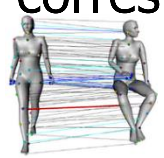
✓ Correspondence algorithms for (nearly) isometric *



complete shapes collections. //not in thesis (done during post-doc).

PG

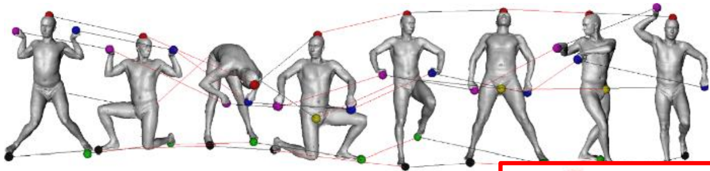
✓ Coarse correspondences:



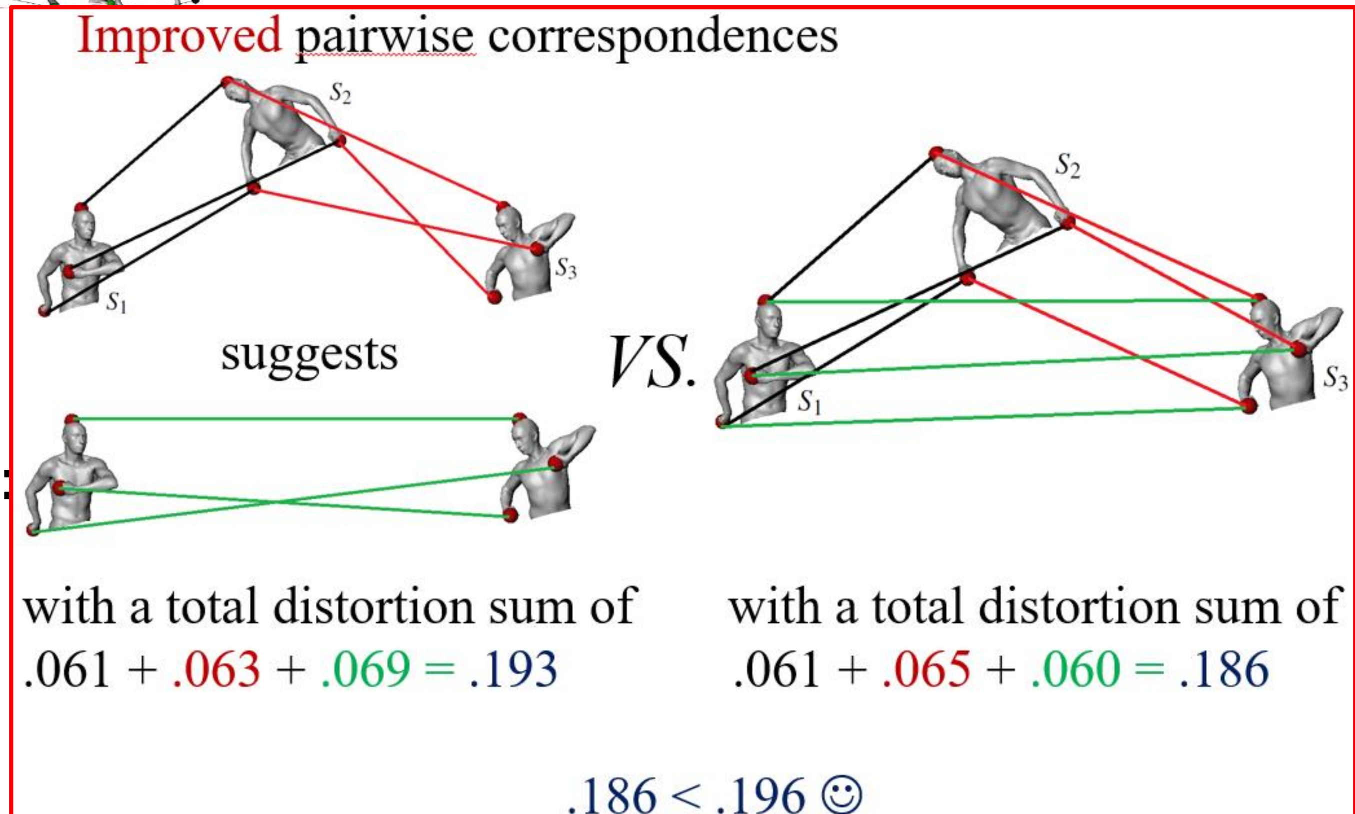
✓ Dense correspondences.

Scope

✓ Correspondence algorithms for (nearly) isometric *

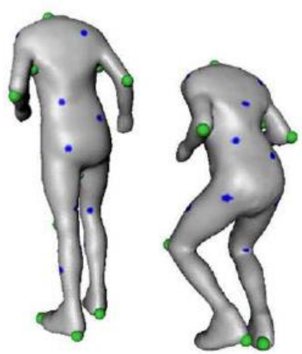


Idea for multiple shape correspondence:

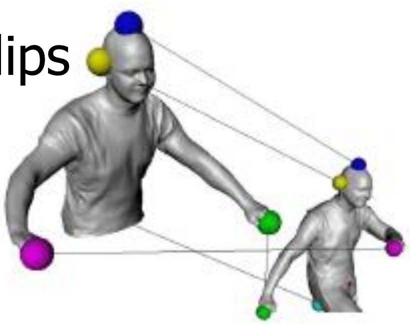


Problems

- ✓ Complete shape correspondence at
  - ✓ coarse resolution
    - ✓ joint sampling

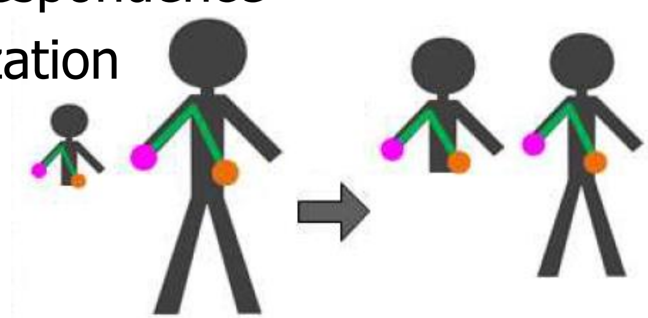


- ✓ symmetric flips

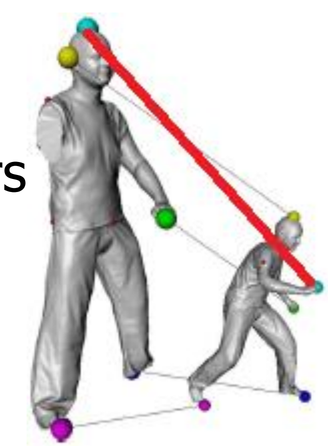


  - ✓ dense resolution
    - ✓ timing

- ✓ Partial shape correspondence
  - ✓ scale normalization



- ✓ outliers



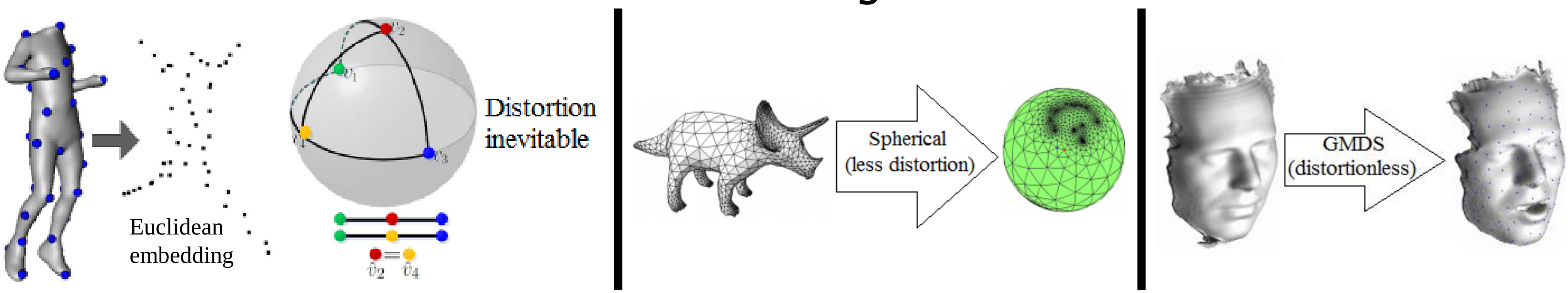
All Algorithms in a Nutshell

Input scenario	Output resolution	Solution paradigm	Computational complexity	Publication
Isometric or nearly isometric	Coarse	Greedy optimization	$O(NV \log V)$	CVPR
Isometric or nearly isometric	Coarse	Greedy optimization and EM algorithm	$O(NV \log V)$	PAMI
Isometric or nearly isometric	Coarse or dense	Combinatorial	$O(V \log V)$	SGP/CGF
Isometric or nearly isometric	Coarse or dense	Combinatorial (with symmetric flip care)	$O(V \log V)$	CGF
Isometric or nearly isometric or partially isometric	Coarse or dense	Combinatorial (part matching)	$O(\binom{N}{M} M!M^3)$	PG/CGF
Isometric or nearly isometric or partially isometric	Coarse or dense	Combinatorial (most general setting)	$O(N^3V \log V)$	CGF

✓ V: # vertices in the original mesh, N: # samples at coarse resolution, $M=5$.

Contributions

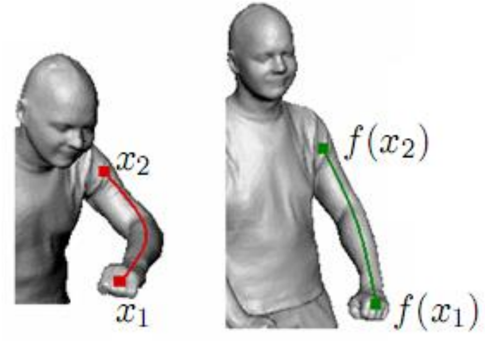
- ✓ Sampling algorithms.
  - ✓ COES, coarse-to-fine, and two extremity sampling methods.
- ✓ Isometric distortion without embedding.



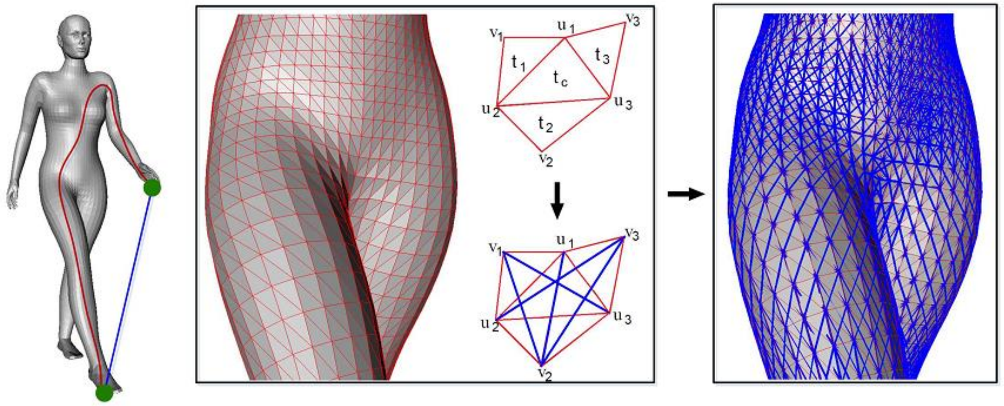
- ✓ Distortion minimization by well-established paradigms.
  - ✓ Graph matching, greedy optimization, EM algo, combinatorial optimization.
- ✓ Map tracking to handle the symmetric flip problem.
- ✓ Dense correspondence w/ the lowest time complexity.
- ✓ Correspondences that are partial and dense at the same time.
- ✓ Partial correspondence in the most general setting.
- ✓ No restriction on topology and triangulation type.

Global Similarity: Isometry

- ✓ All of our methods are purely isometric.
- ✓ Similar shapes have similar metric structures.

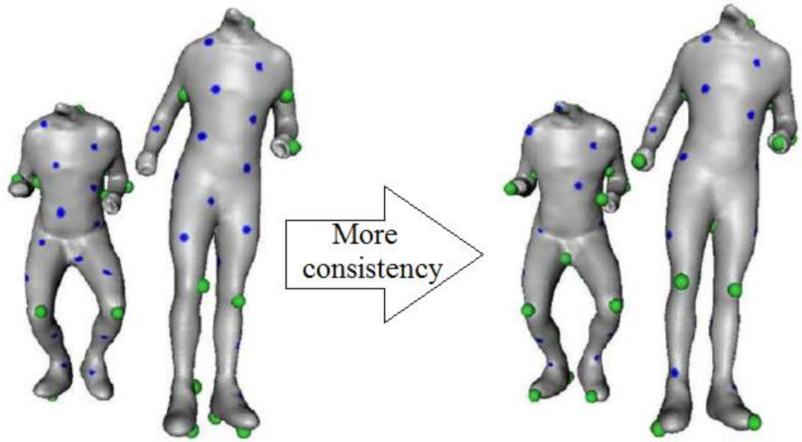


  - ✓ Metric: geodesic distance (in use) vs. diffusion-based distances.

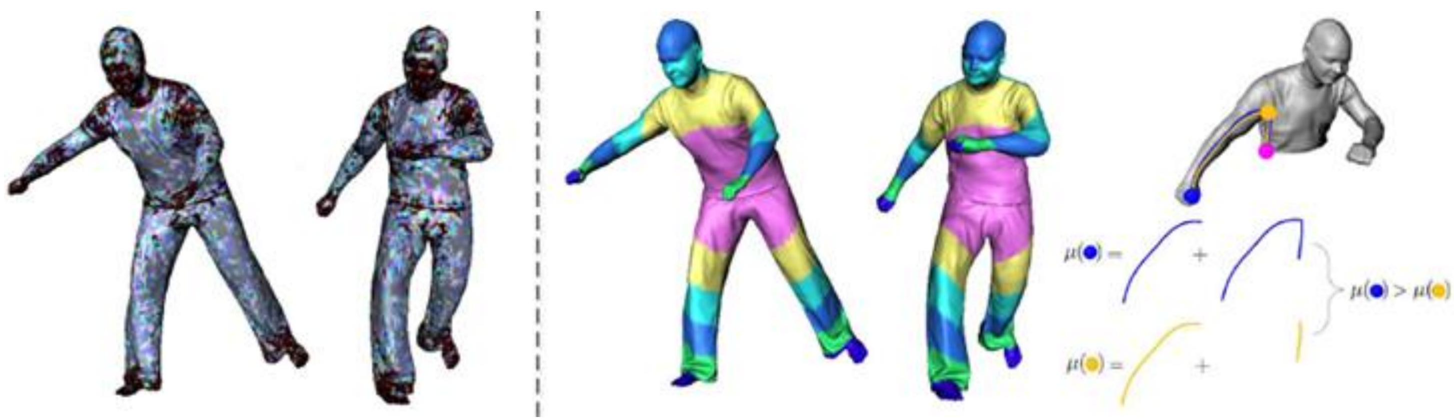


Local Similarity: Descriptors

✓ More consistent joint-sampling which helps matching.

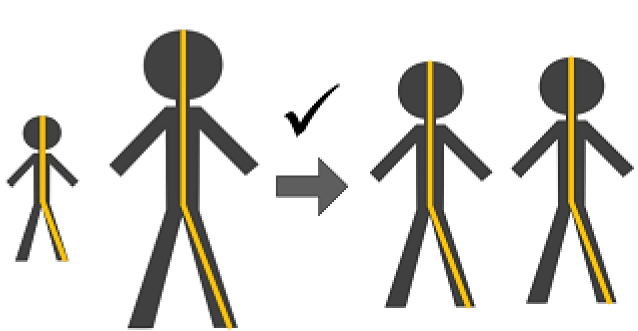


✓ Gaussian curvatures and average geodesic distances in use.

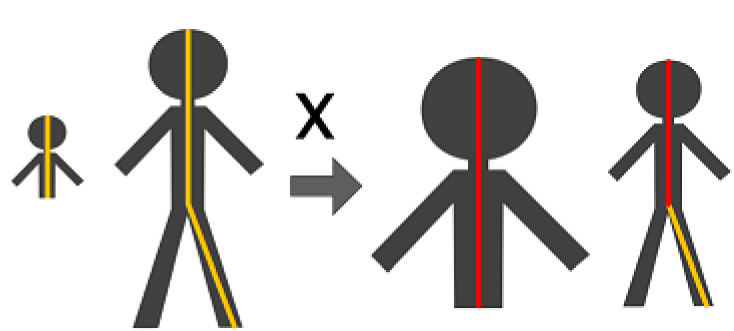


Scale Normalization

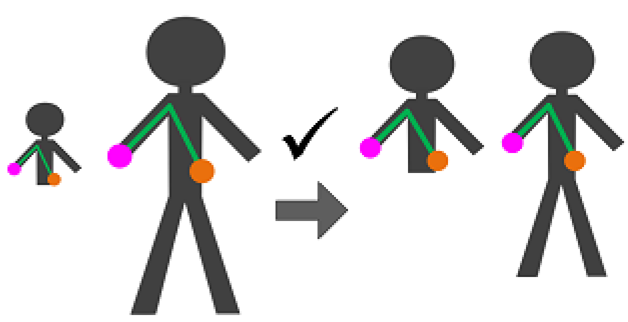
- ✓ Scale normalization to prepare geodesic distances for upcoming isometric distortion computations.



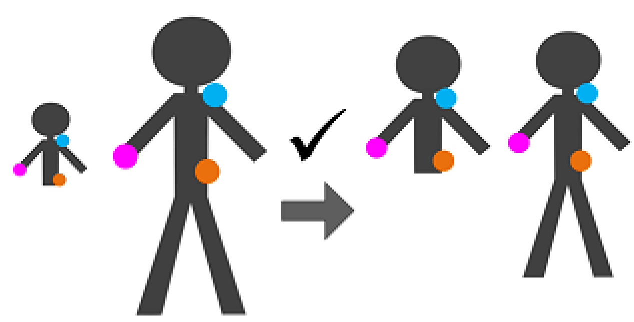
Complete shapes (scale by max geodesic)



Partial shapes (max geodesic based normalization fails)



Partial shapes (scale by trusted matches)



Partial shapes (scale by Euclidean embedding, e.g., Möbius)

Isometric Distortion

- ✓ Given $\S : S \rightarrow T$, measure its isometric distortion:

$$D_{\text{iso}}(\S) = \frac{1}{|\S|} \sum_{(s_i, t_j) \in \S} d_{\text{iso}}(s_i, t_j)$$

$$d_{\text{iso}}(s_i, t_j) = \frac{1}{|\S'|} \sum_{(s_l, t_m) \in \S'} |g(s_i, s_l) - g(t_j, t_m)|$$

$\S' = \S - \{(s_i, t_j)\}$ in the most general setting.

$g(.,.)$: normalized geodesic distance b/w two vertices.

- ✓ $O(N^2)$ for a map of size N.

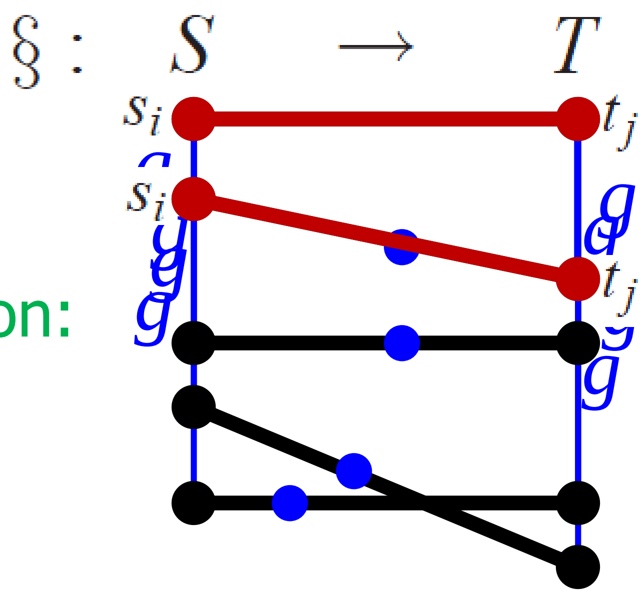
Isometric Distortion Illustration

$$D_{\text{iso}}(\S) = \frac{1}{|\S|} \sum_{(s_i,t_j)\in \S} d_{\text{iso}}(s_i,t_j)$$

$$d_{\text{iso}}(s_i,t_j) = \frac{1}{|\S'|} \sum_{(s_l,t_m)\in \S'} |g(s_i,s_l) - g(t_j,t_m)|$$

$$\S' = \S - \{(s_i,t_j)\}$$

in action:



$$\left.\begin{aligned} d_{\text{iso}}(s_i,t_j) &= \;>0 + 0 + >0 + 0 \\ d_{\text{iso}}(s_i,t_j) &= \,.\,. \\ &\vdots \end{aligned}\right\}$$ average for $D_{\text{iso}}(\S)$.

Scale-invariant Isometric Distortion

- ✓ Given $\S : S \rightarrow T$, measure its scale-inv. isometric distortion:

$$\mathcal{D}(\S) = \sum_{(s_i,t_j)\in\S} \sum_{\substack{(s_a,t_b)\in\S \\ (s_c,t_d)\in\S}} |\rho(s_i,t_j;s_a,t_b) - \rho(s_i,t_j;s_c,t_d)|$$

$$\text{ratio function } \rho(s_i,t_j;s_k,t_l) = \max\left(\frac{g(s_i,s_k)}{g(t_j,t_l)}, \frac{g(t_j,t_l)}{g(s_i,s_k)}\right)$$

$g(.,.)$: unnormalized/raw geodesic distance b/w two vertices.

- ✓ This measure based on raw geodesics provides few trusted matches to be used in scale normalization.
- ✓ $O(N^3)$ for a map of size N.

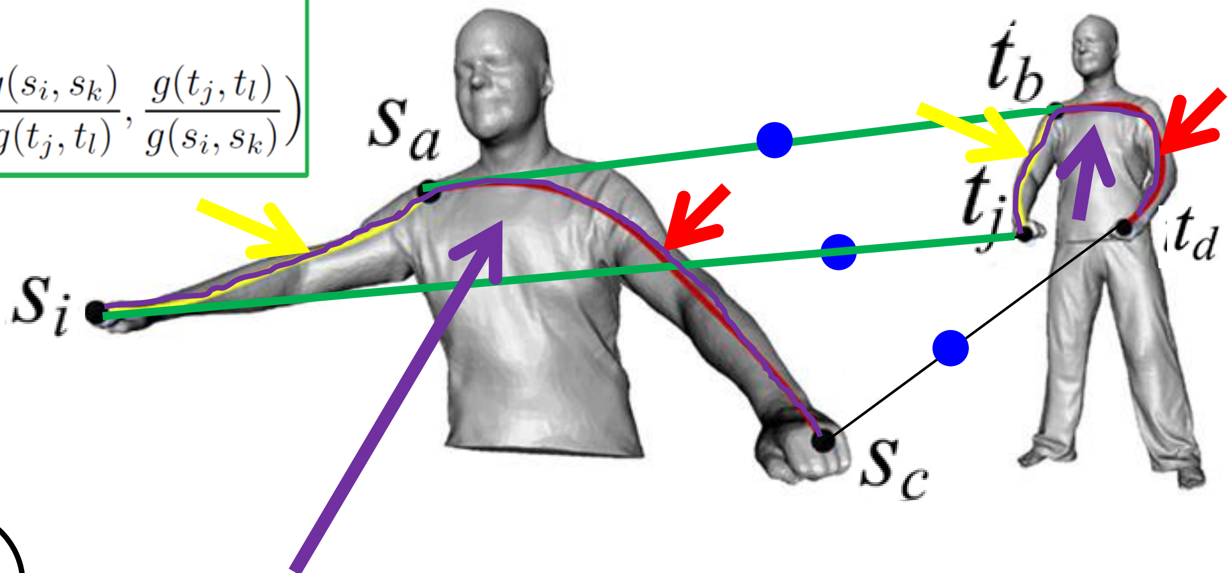
Scale-inv. Isometric Distortion Illustration

$$\mathcal{D}(\S) = \sum_{(s_i,t_j)\in\S} \sum_{\substack{(s_a,t_b)\in\S \\ (s_c,t_d)\in\S}} |\rho(s_i,t_j;s_a,t_b) - \rho(s_i,t_j;s_c,t_d)|$$

ratio function $\rho(s_i,t_j;s_k,t_l) = \max\left(\frac{g(s_i,s_k)}{g(t_j,t_l)}, \frac{g(t_j,t_l)}{g(s_i,s_k)}\right)$

in action:

$\S : S \rightarrow T$



$$d_{\text{iso}}(s_i,t_j) = 0$$
$$d_{\text{iso}}(s_i,t_j) = 0$$
$$\vdots$$

average for $\mathcal{D}(\S)$.

Minimizing Isometric Distortion

$$\boxed{\S^* = \arg\min_{\S} D_{\text{iso}}(\S)}$$

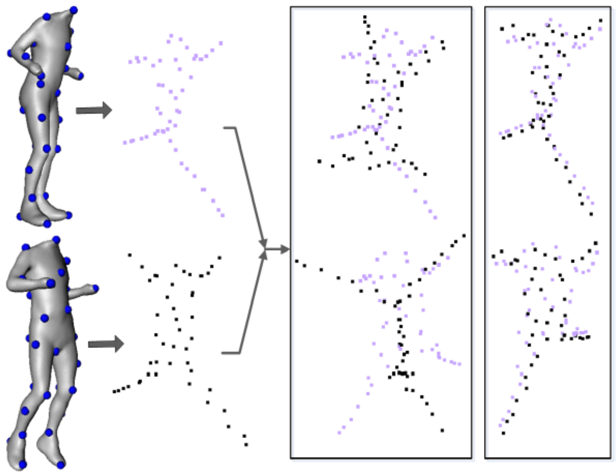
- ✓ Optimization by
  - ✓ Greedy (CVPR'10).
  - ✓ EM framework (PAMI).
  - ✓ Combinatorial in C2F fashion (SGP'11, CGF'13).
  - ✓ Rank-and-vote-and-conquer (CGF'14).

$$\boxed{\S^* = \arg\min_{\S} \mathcal{D}_{\text{iso}}(\S)}$$

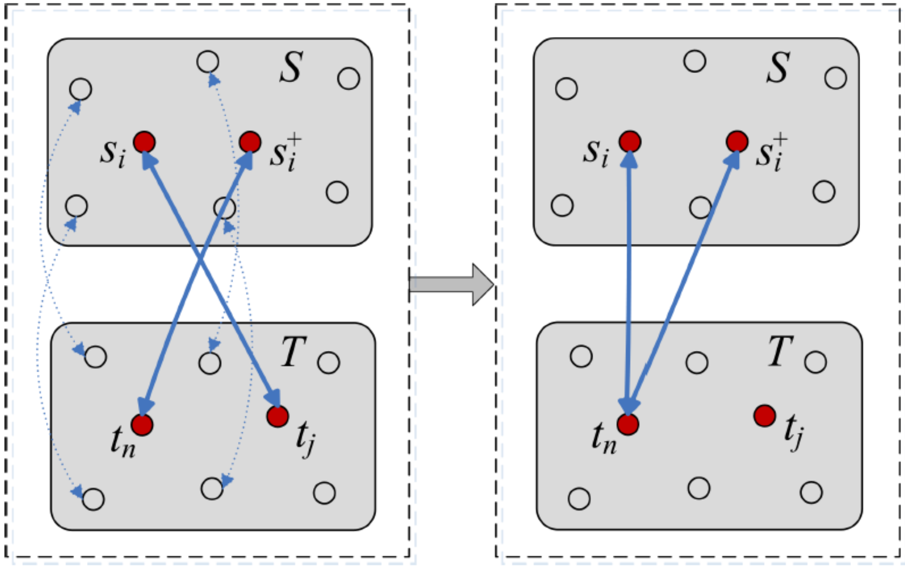
- ✓ Optimization by
  - ✓ Combinatorial (PG'12).

Greedy Optimization

- ✓ Initialization by spectral embedding & alignment.



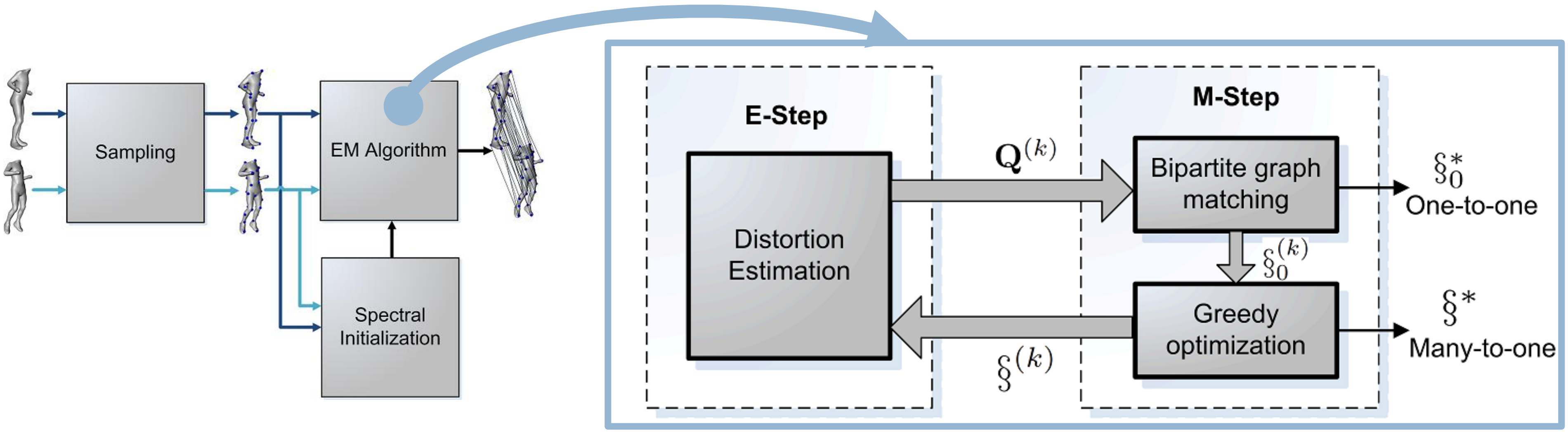
- ✓ Refinement by greedy optimization.



Yusuf Sahillioğlu and Yücel Yemez, 3D Shape Correspondence by Isometry-Driven Greedy Optimization, *Proc. Computer Vision and Pattern Recognition (CVPR)*, pp. 453-458, 2010.

EM Framework

✓ Refine initial spectral correspondence even further in EM framework.



✓ Minimization of the isometric distortion = Maximization of the log-likelihood function encoded in matrix $\mathbf{Q}$: probability of matching source s_i to target t_j.

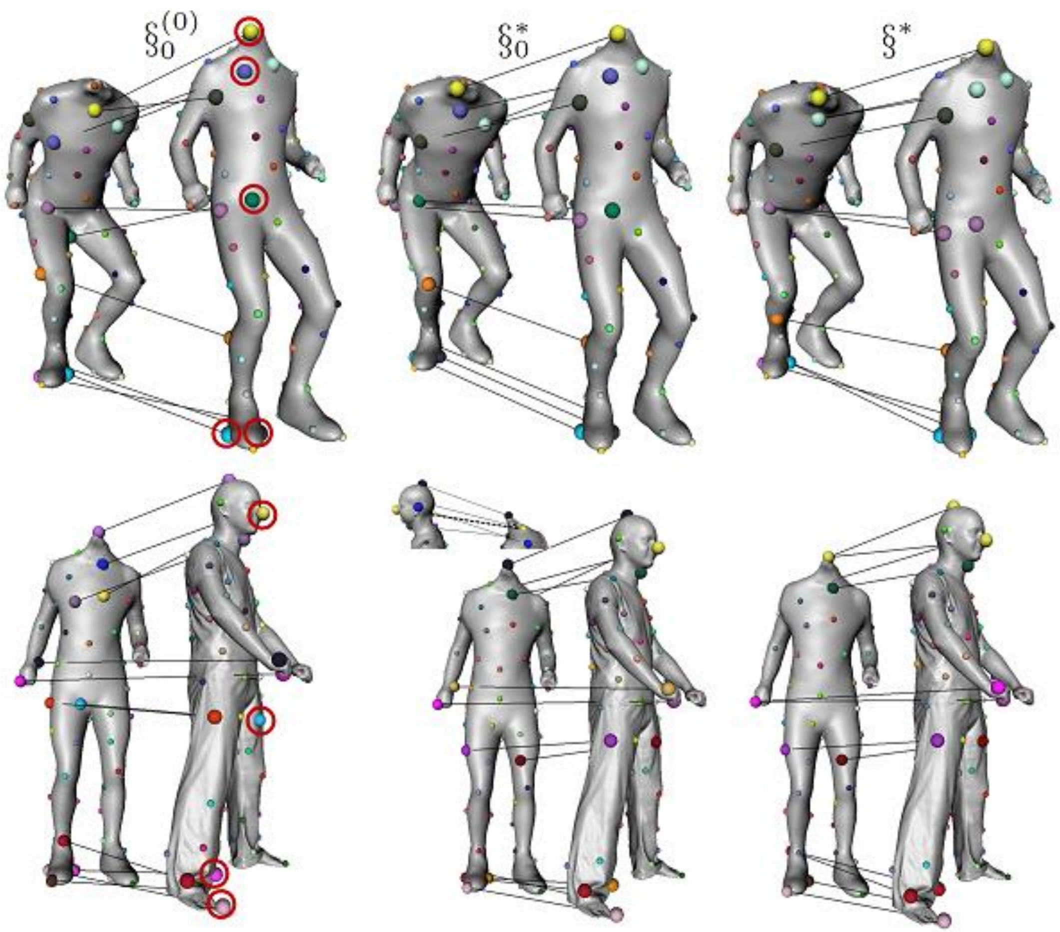
✓ $q_{ij} = \mathrm{P}(t_j | s_i) = \frac{1}{T_i} e^{-\beta d_{\text{iso}}(s_i, t_j)}$

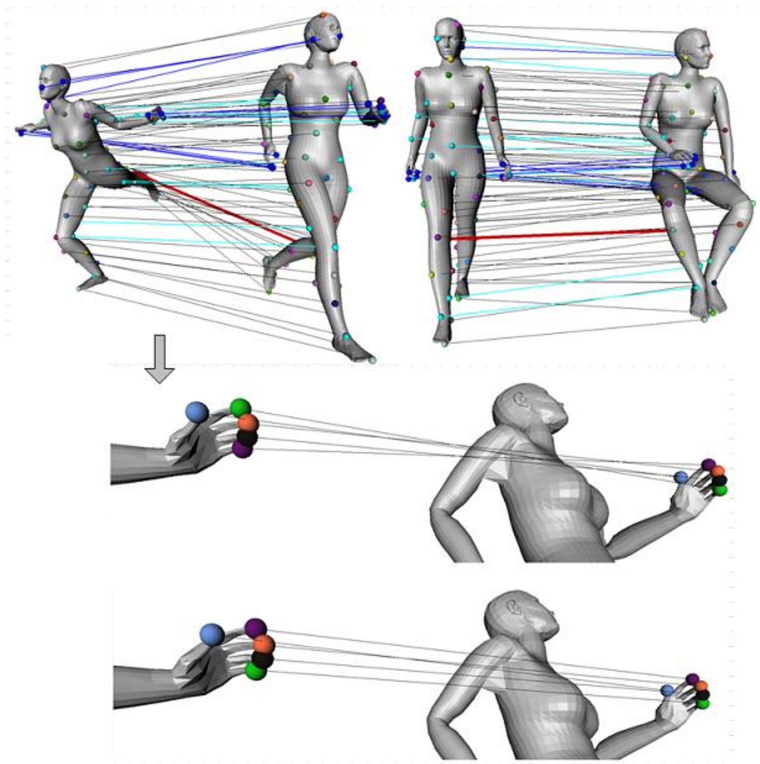
✓ $\boxed{\S^* = \arg\max_{\S} \log \mathrm{P}(\S | \mathcal{X}, \mathbf{Q})}$

Yusuf Sahillioğlu and Yücel Yemez, Minimum-Distortion Isometric Shape Correspondence Using EM Algorithm, *PAMI*, to appear, 2012.

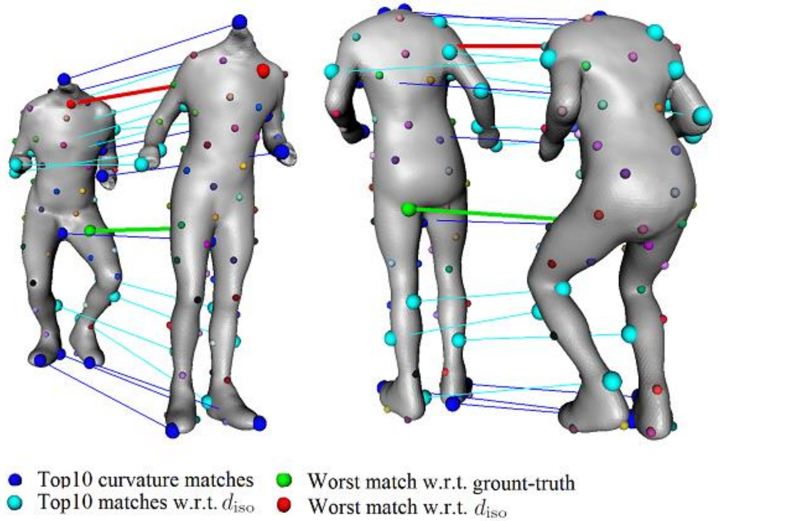
EM Framework (Results)

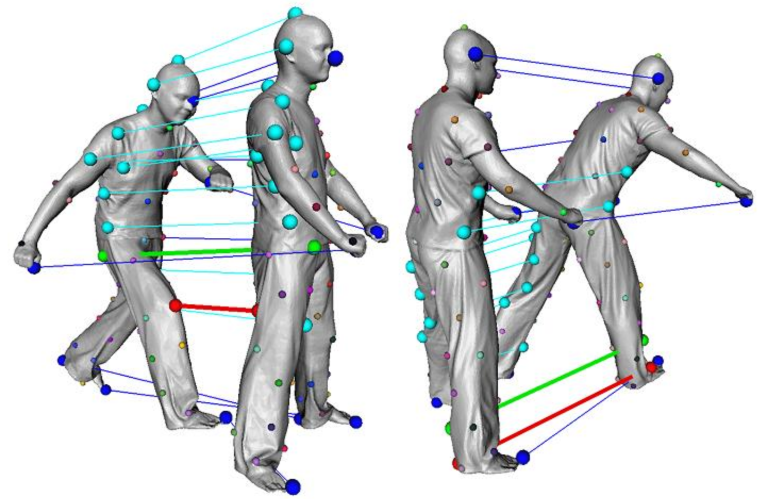
✓ Initial spectral correspondence refined (one-to-one and many-to-one maps).

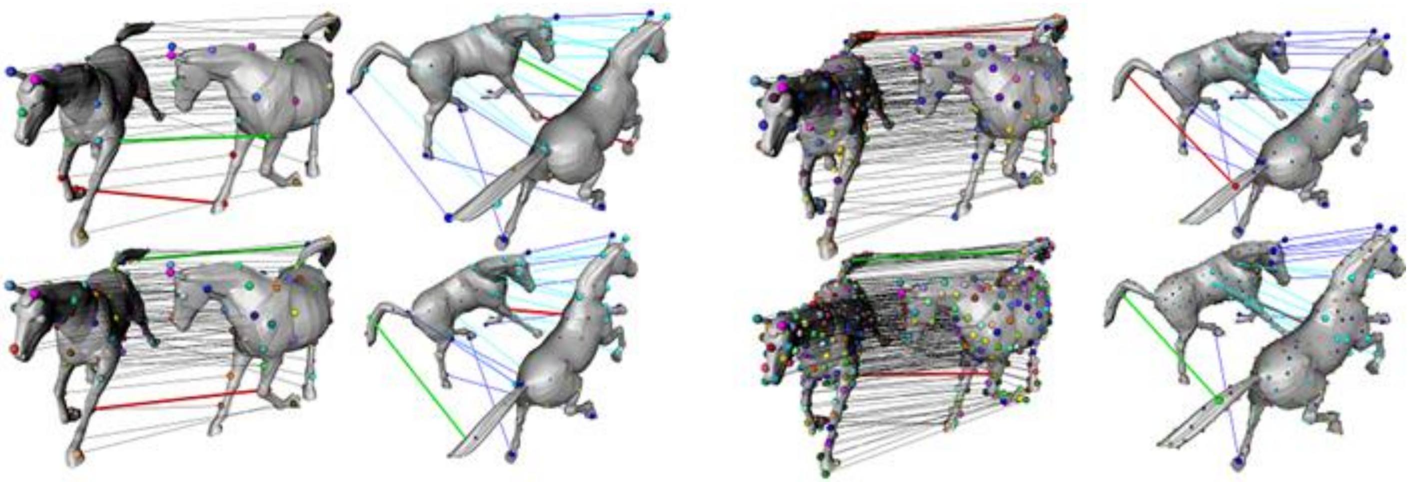




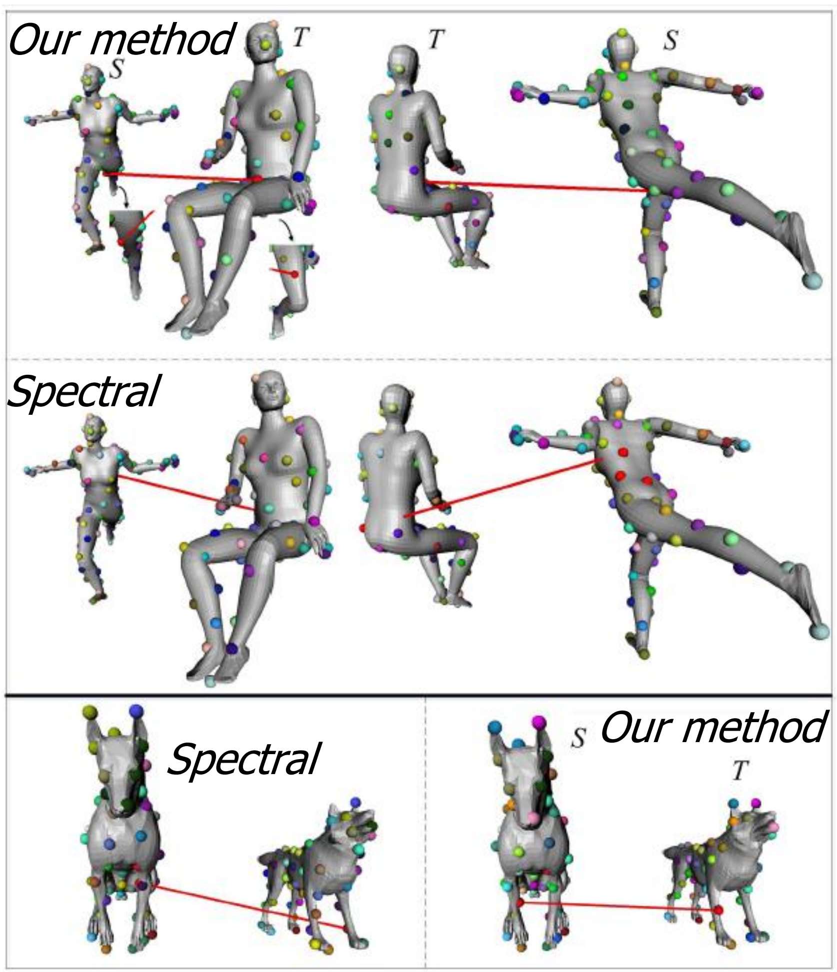
EM Framework (Results)



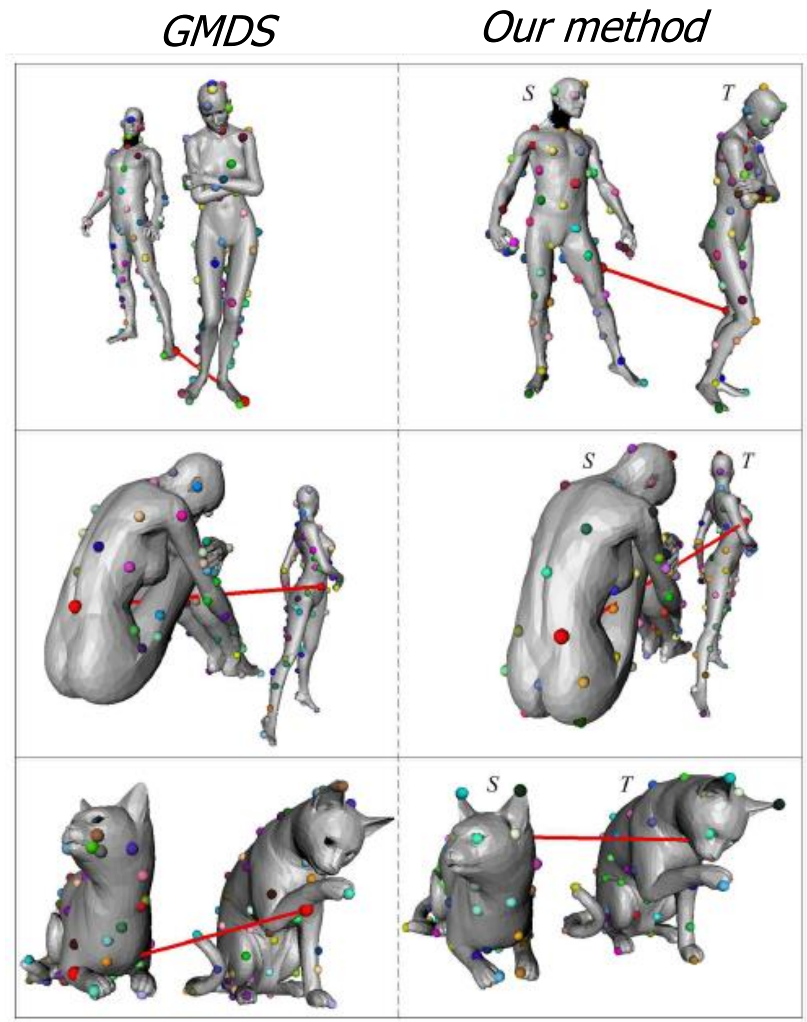




EM Framework (Comparisons)



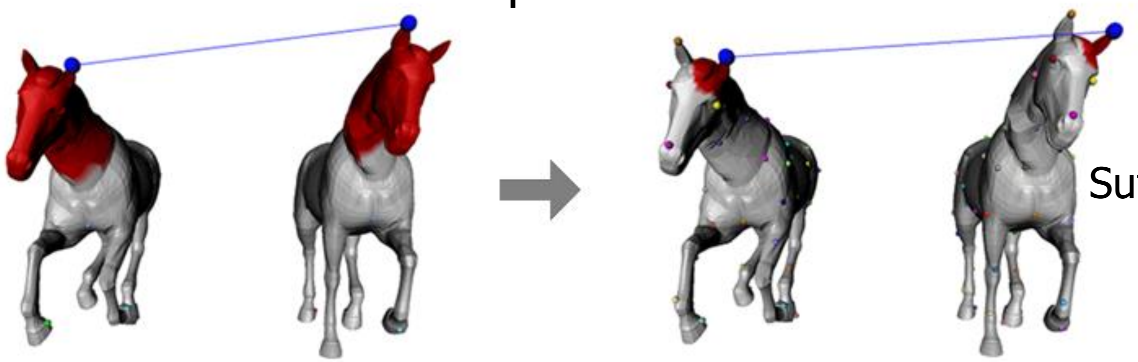
Spectral : worse worsts, missing salient points.



GMDS : clustered matches, missing salient pnts.

EM Framework (Limitations)

1. Mismatches due to lack of samples.



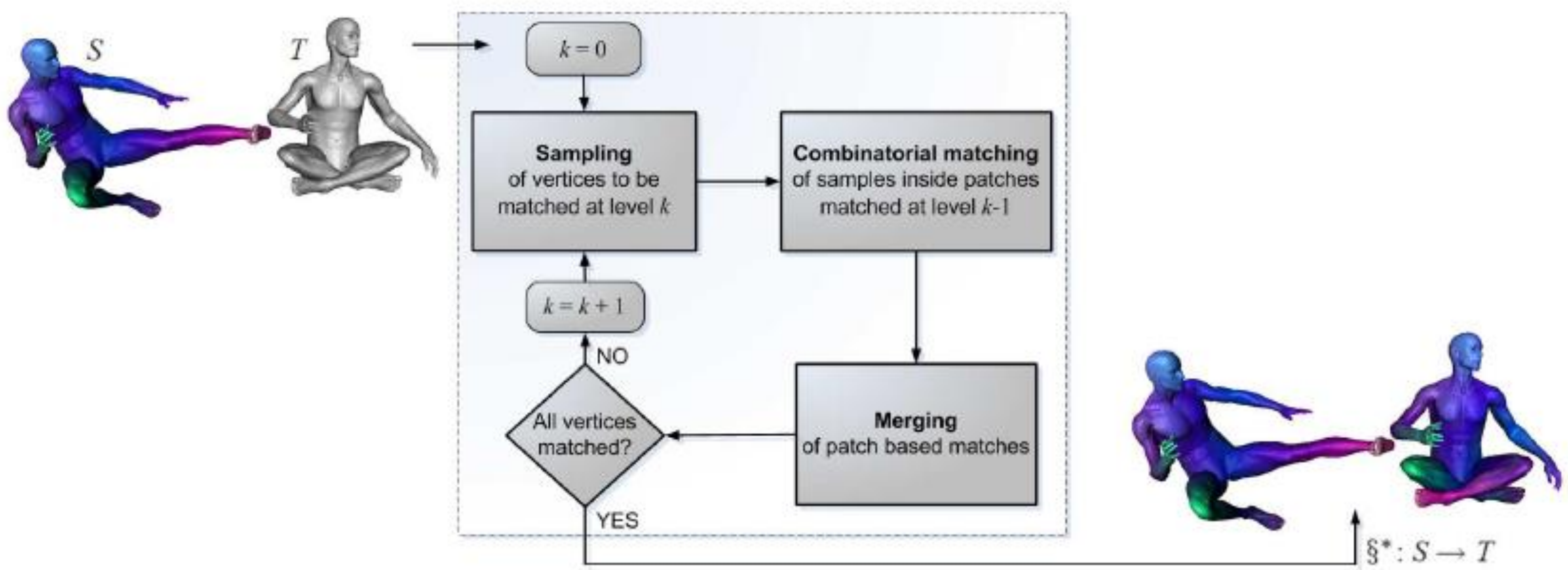
Sufficient # of samples.

2. No efficient extension to dense correspondence due to cubic EM framework.
3. No caution for symmetric flips.
4. No support for partially isometric shapes.

- ✓ Limitation 1 handled by adjusting sampling distance parameter or in coarse-to-fine (C2F) fashion as proposed in SGP w/o any user interaction.
- ✓ Limitation 2 handled by SGP which is less accurate than this in achieving sparse correspondences.
- ✓ Limitation 3 handled by CGF extension of SGP.
- ✓ Limitation 4 handled partially by PG and fully by SIGGRAPH Asia.

C2F Combinatorial Optimization

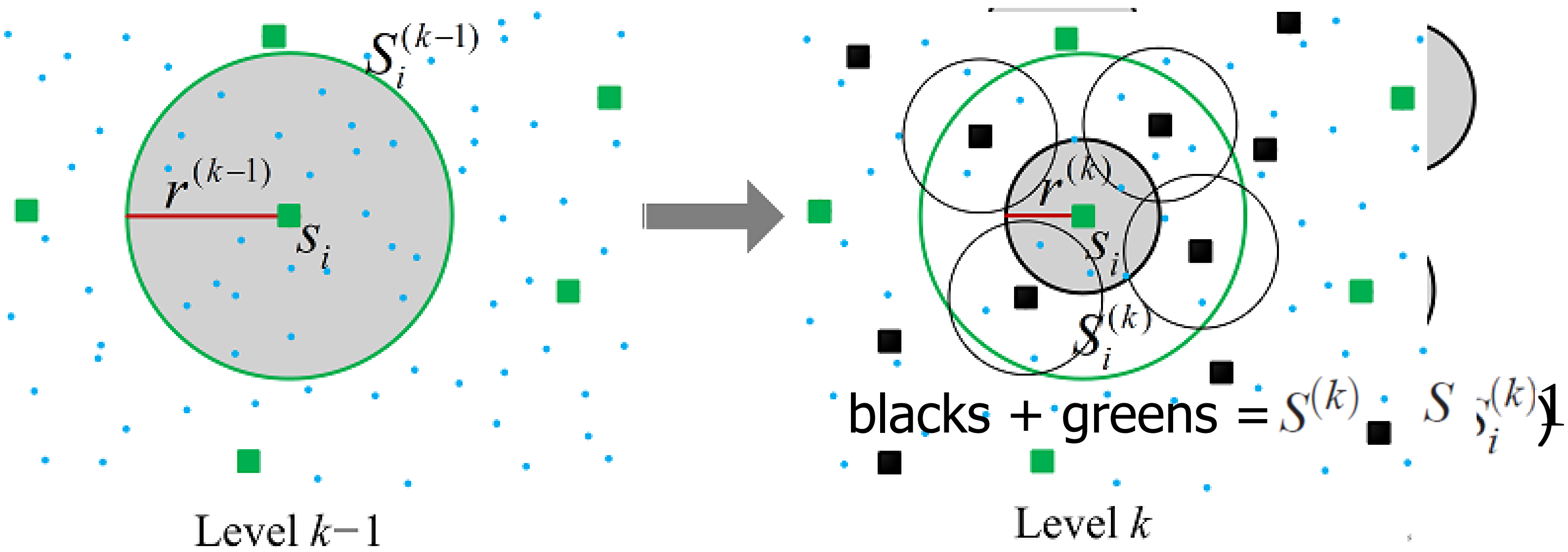
- ✓ Optimal mapping maps nearby vertices in source to nearby vertices in target.
  - ✓ Recursively subdivide matched patches into smaller patches (C2F sampling) to be matched (combinatorial search).
  - ✓ That is combinatorial matching in a coarse-to-fine fashion.



Yusuf Sahillioğlu and Yücel Yemez, Coarse-to-Fine Combinatorial Matching for Dense Isometric Shape Correspondence, *Computer Graphics Forum (SGP)*, Vol. 30, No. 5, pp. 1461-1470, 2011.

C2F Combinatorial Optimization

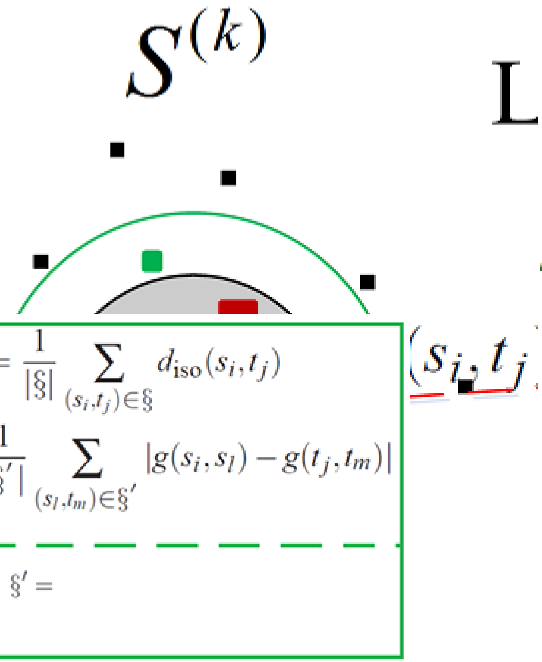
✓ C2F sampling.



Yusuf Sahillioğlu and Yücel Yemez, Coarse-to-Fine Combinatorial Matching for Dense Isometric Shape Correspondence, *Computer Graphics Forum (SGP)*, Vol. 30, No. 5, pp. 1461-1470, 2011.

C2F Combinatorial Optimization

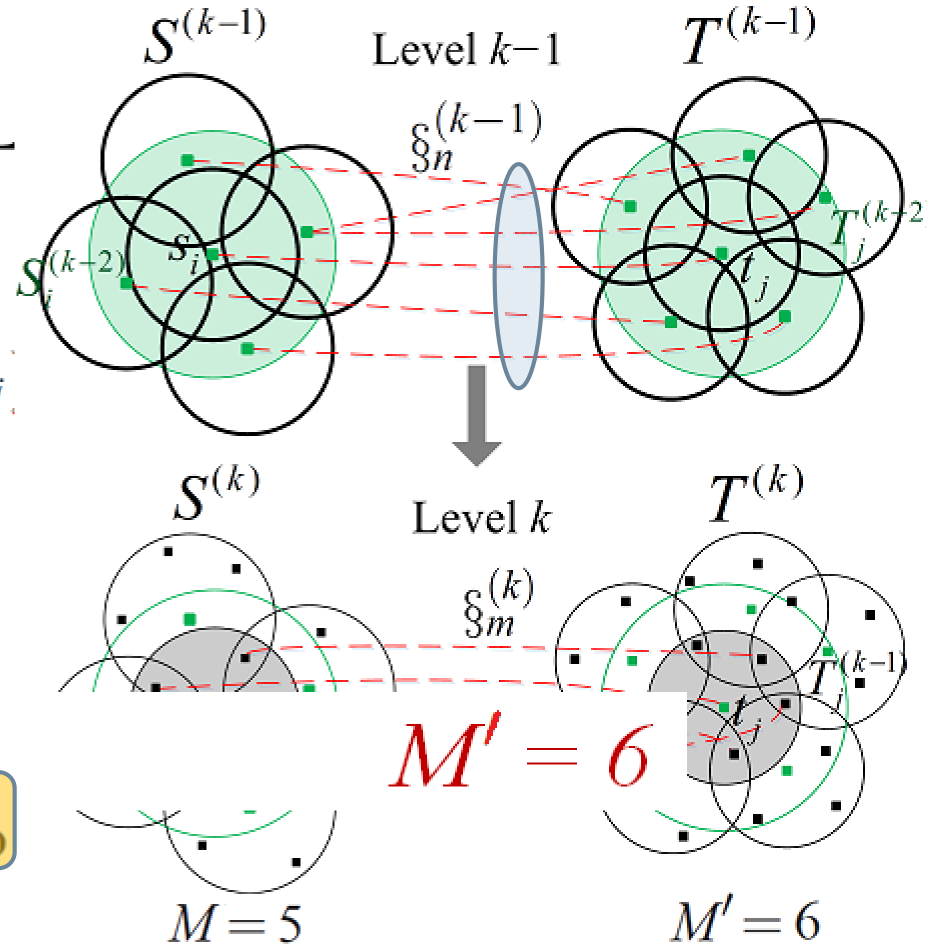
✓ Combinatorial matching.



$$D_{\text{iso}}(\S) = \frac{1}{|\S|} \sum_{(s_i,t_j)\in\S} d_{\text{iso}}(s_i,t_j)$$

$$d_{\text{iso}}(s_i,t_j) = \frac{1}{|\S'|} \sum_{(s_l,t_m)\in\S'} |g(s_i,s_l) - g(t_j,t_m)|$$

$\S' =$



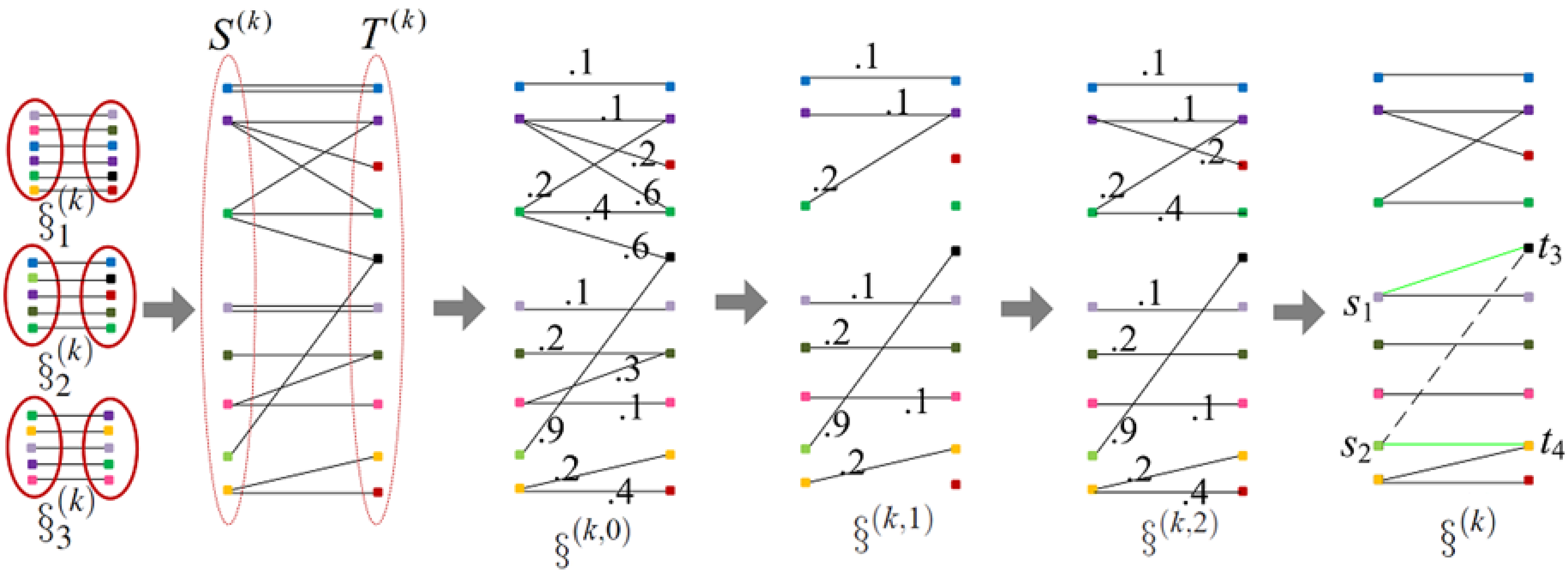
$M = 5$ $M' = 6$

$\binom{M'}{M} M!$ evaluations of D_{iso} $S^{(k)}$

with $\S' = \S \cup \bigcup_n \S_n^{(k-1)}$

C2F Combinatorial Optimization

✓ Merging patch-to-patch correspondences into one correspondence over the whole surface.



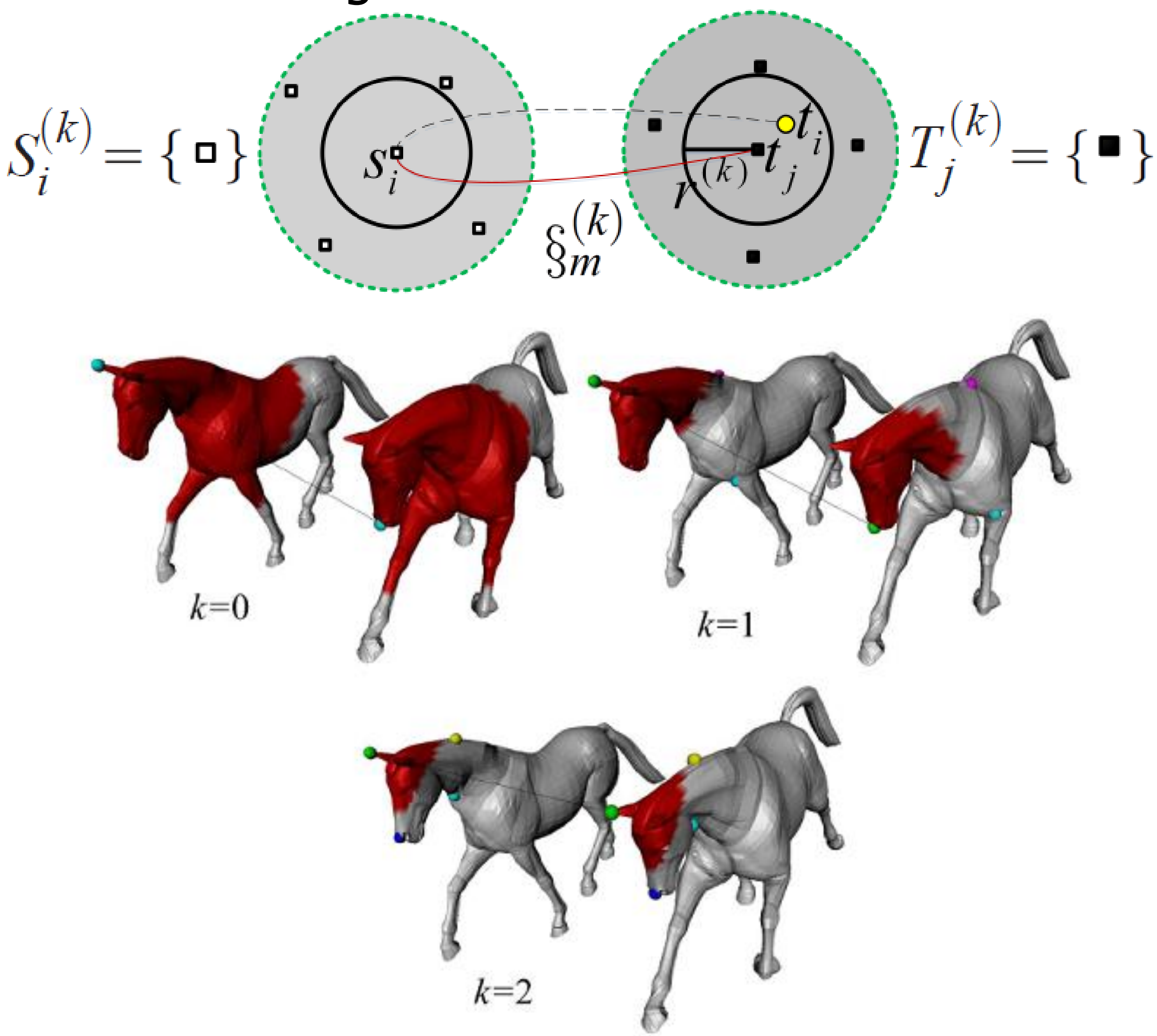
1st pass over stage samples to assign only one match per isolated sample, the one with the min d_{iso}.

2nd pass over stage samples to assign only one match per isolated sample, the one with the min d_{iso}.

Multi-matches with $d_{iso} > 2D$ also i.e. outliers made available

C2F Combinatorial Optimization

- ✓ Inclusion assertion for algorithm correctness.



C2F Combinatorial Optimization

- ✓ Saliency sorting.

$$O(N\log N) \text{ where } N = \max(|S|,|T|)$$

- ✓ C2F sampling.
  - ✓ Restricted to the patch to be sampled, Dijkstra's shortest paths takes:

$$O\Big(\sum_{\boxed{k}=1}^{K} \underbrace{\cancel{M^k}}_{\#\text{ patches}} \cdot \frac{N}{\cancel{M^k}}\log_M \underbrace{\frac{N}{M^k}}_{\text{sizeof(patch)}}\Big) = O\Big(\underbrace{\overbrace{N\cdot\log_M\frac{M^K}{M^{\boxed{K}}}}^{k=K} + \overbrace{N\cdot\log_M\frac{M^K}{M^{\boxed{K-1}}}}^{k=K-1} + \cdots + \overbrace{N\cdot\log_M\frac{M^K}{M^{\boxed{1}}}}^{k=1}}_{\substack{K = \log_M N \\ \text{since } N = M^K.}}\Big)$$

$$= O(N\log N)$$

C2F Combinatorial Optimization

✓ Patch-based combinatorial matching.

$O(\sum_{k=1}^{K} \underset{\text{\# patches}}{\underset{\downarrow}{M^k}} \cdot M!)$ because each pair is matched in $O(M!)$ time.

$$= O(\underbrace{\overset{\boxed{k=K}}{M^{\boxed{K}}} \cdot M! + \overset{\boxed{k=K-1}}{M^{\boxed{K-1}}} \cdot M! + \cdots + \overset{\boxed{k=1}}{M^{\boxed{1}}} \cdot M!}_{K \,=\, \log_M N}) = O(N \log N) \text{ since } M^K = N.$$

✓ Merging.

M^k: size of the mapping at level k.

E: evenly-spaced subset of E $(= 100)$ matches as $\S'$.

3-step merging takes $O(\sum_{k=1}^{K} 3 \cdot \underset{d_{\text{iso}} \text{ computations}}{\underset{\downarrow}{M^k}} E) = O(N \log N)$ time.

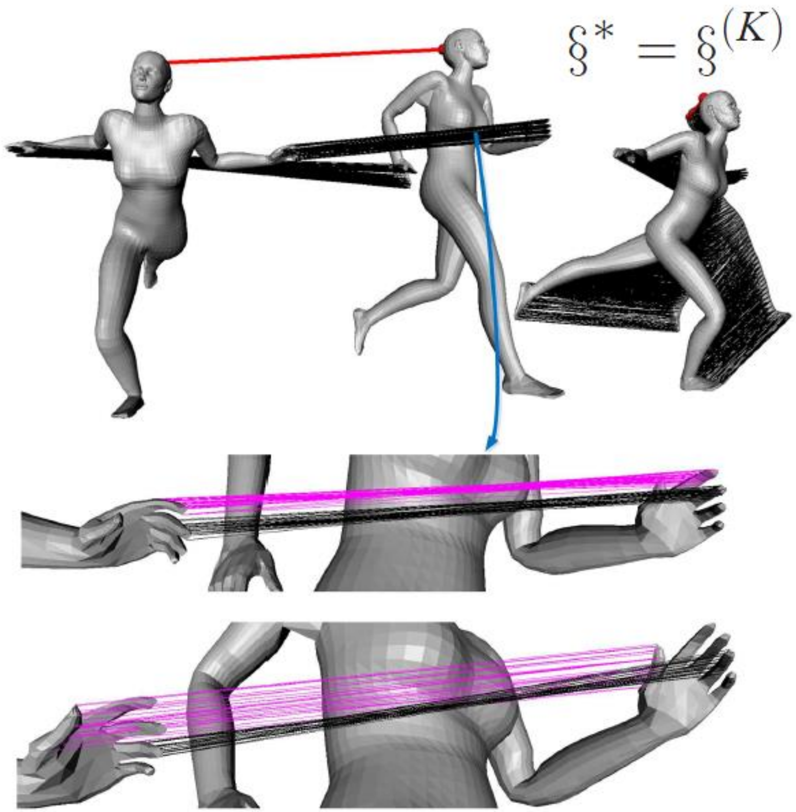
C2F Combinatorial Optimization

- ✓ Overall

$$O(N \log N)$$

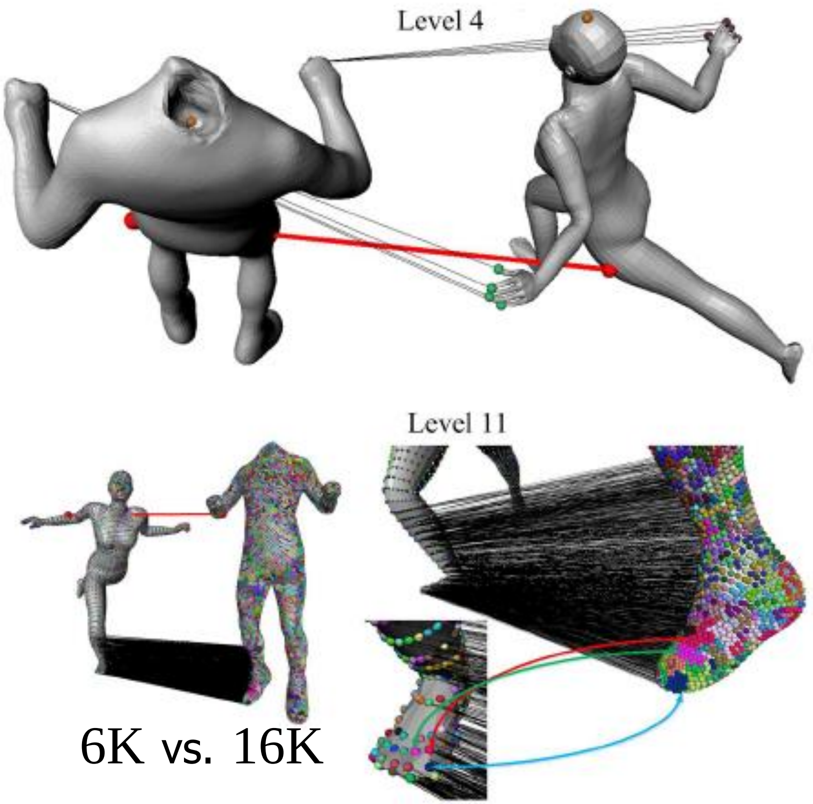
C2F Combinatorial Optimization (Results)

✓ Details captured, smooth flow.



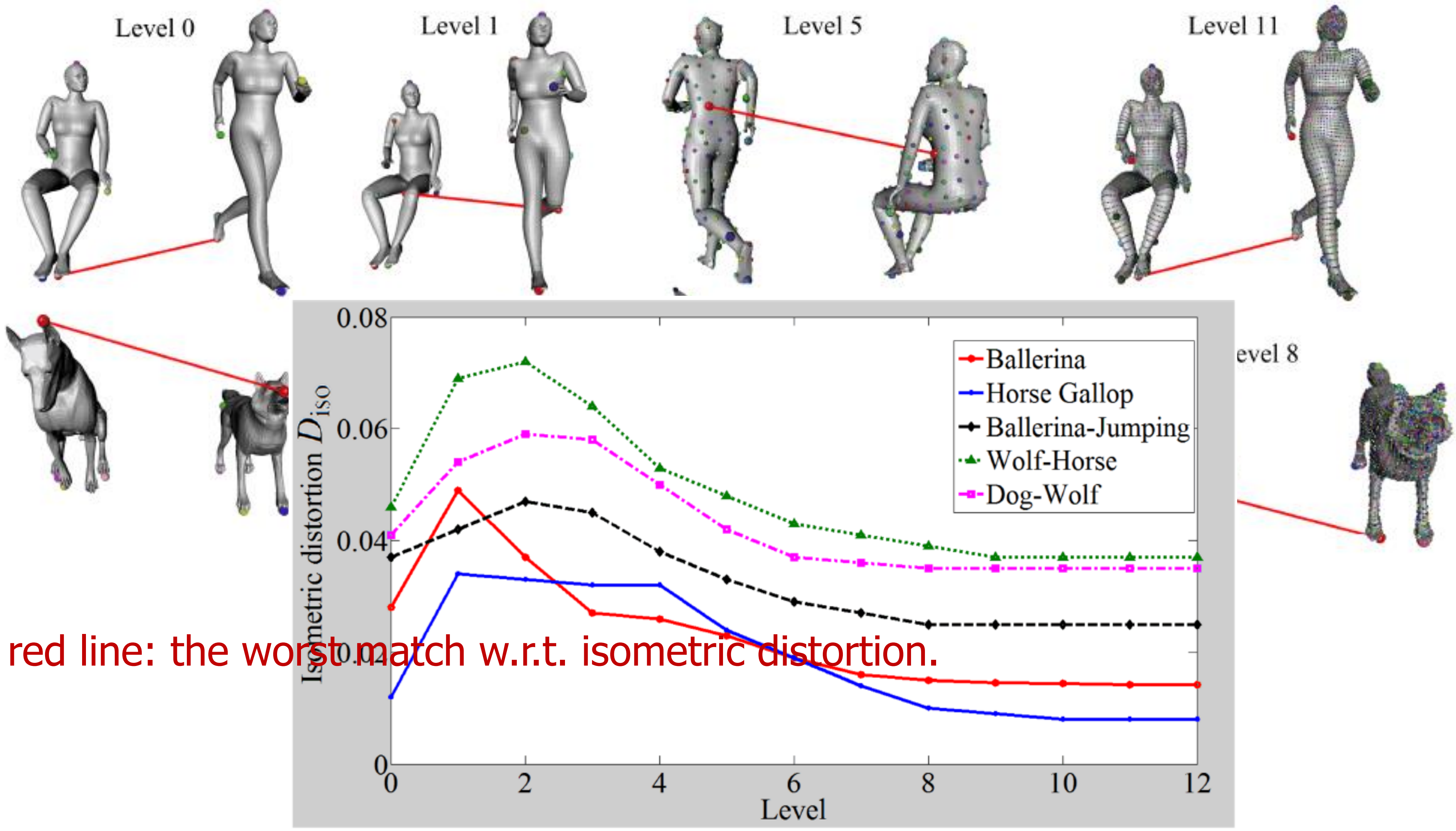
red line: the worst match w.r.t. isometric distortion.

✓ Many-to-one.



✓ Two meshes at different resolutions.

C2F Combinatorial Optimization (Results)

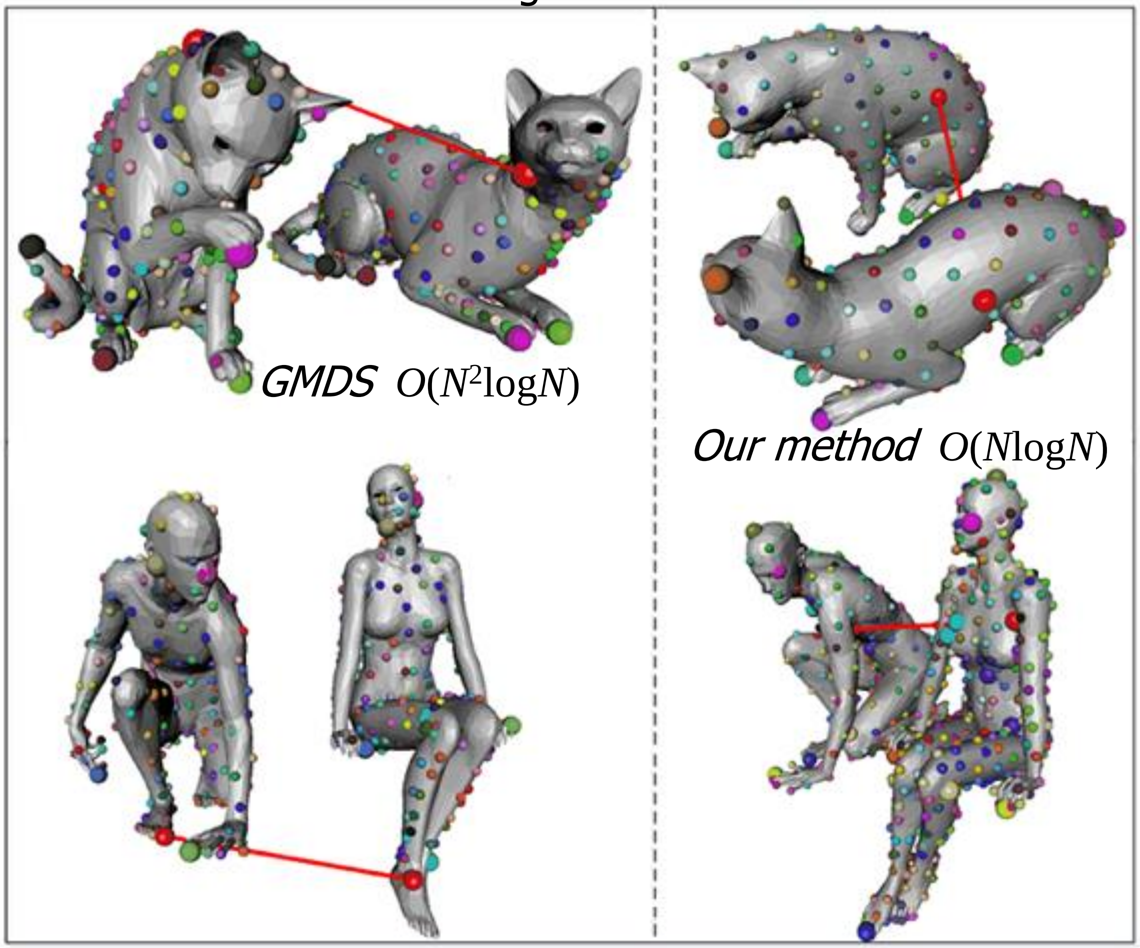


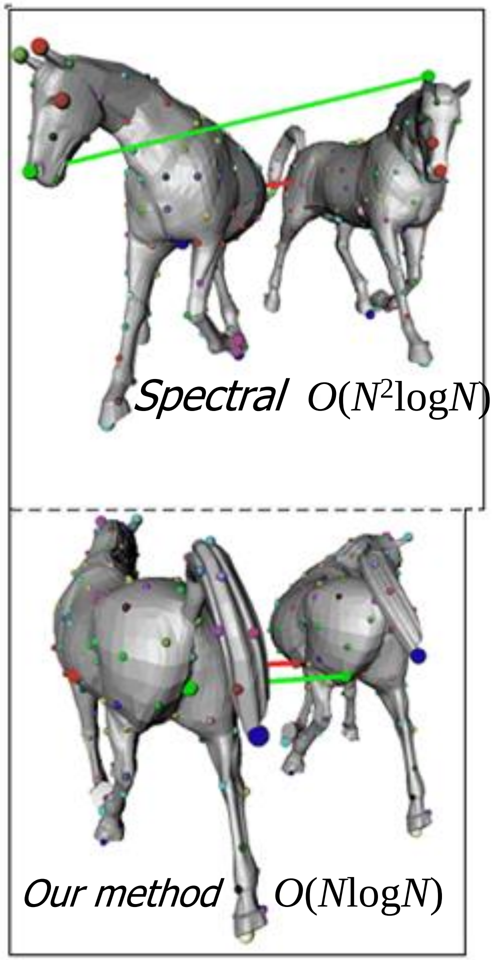
red line: the worst match w.r.t. isometric distortion.

C2F Combinatorial Optimization (Comparisons)

✓ Comparisons.

Nonrigid world dataset





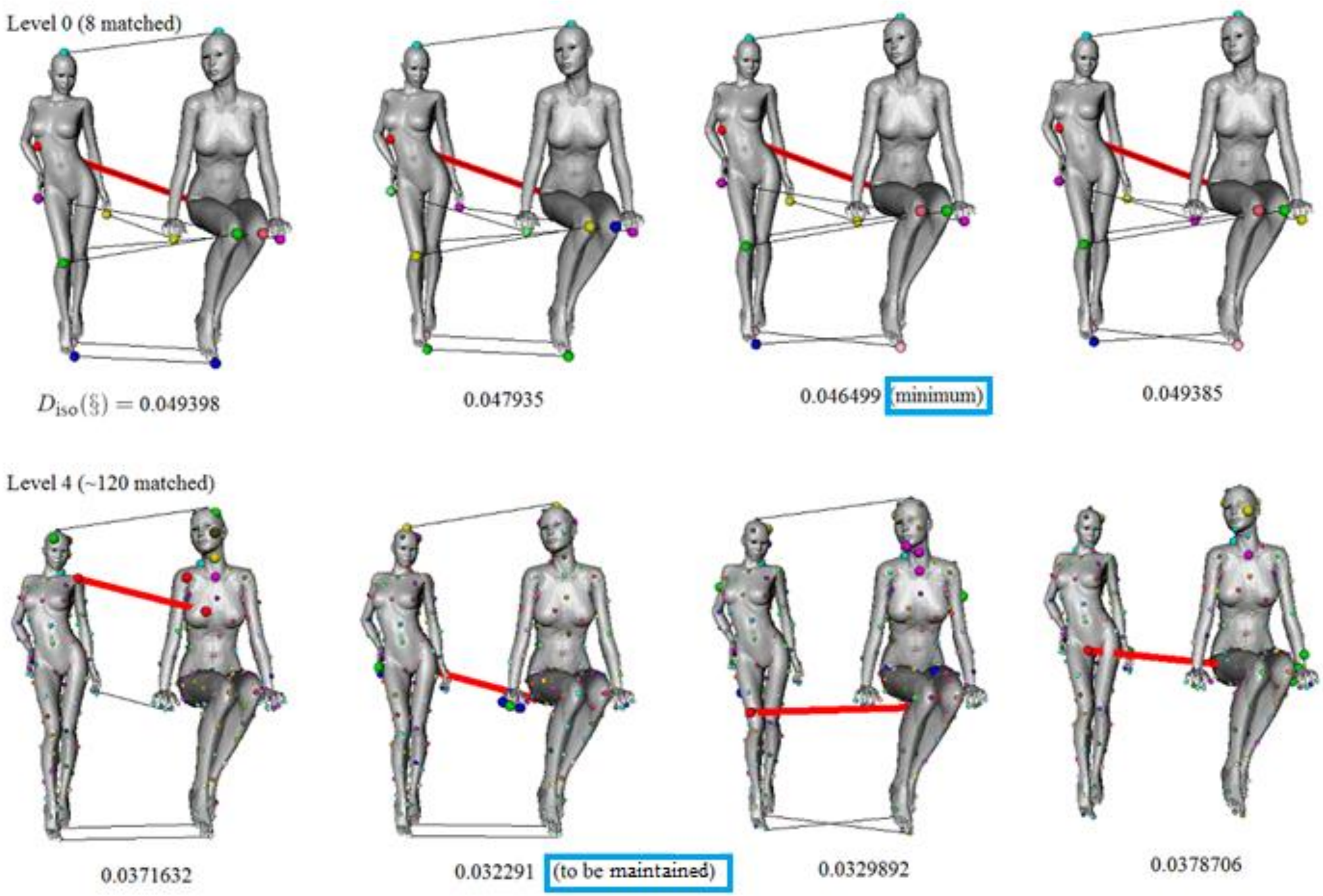
C2F Combinatorial Optimization (Limitation)

- ✓ Symmetric flip problem.
  - ✓ Purely isometry-based methods naturally fail at symmetric inputs.
  - ✓ Due to multiple local minima of non-convex distortion function, our method initialized w/ coarse sampling may fail to find the true *optimum.*

- ✓ Solution is based on map tracking.

C2F Combinatorial Optim. w/ Tracking

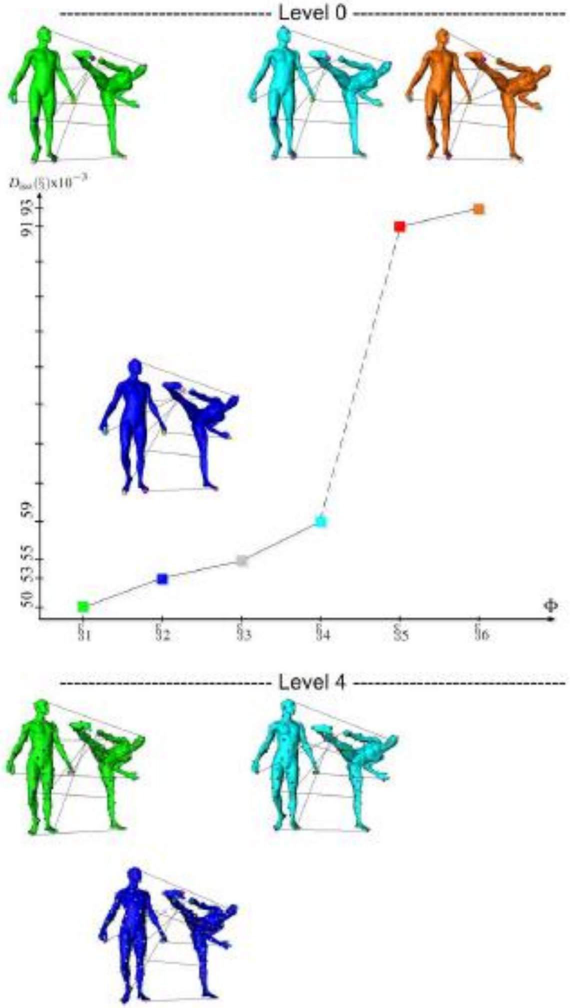
- ✓ Track potential maps decided at level 0 until level 4 and maintain the best.

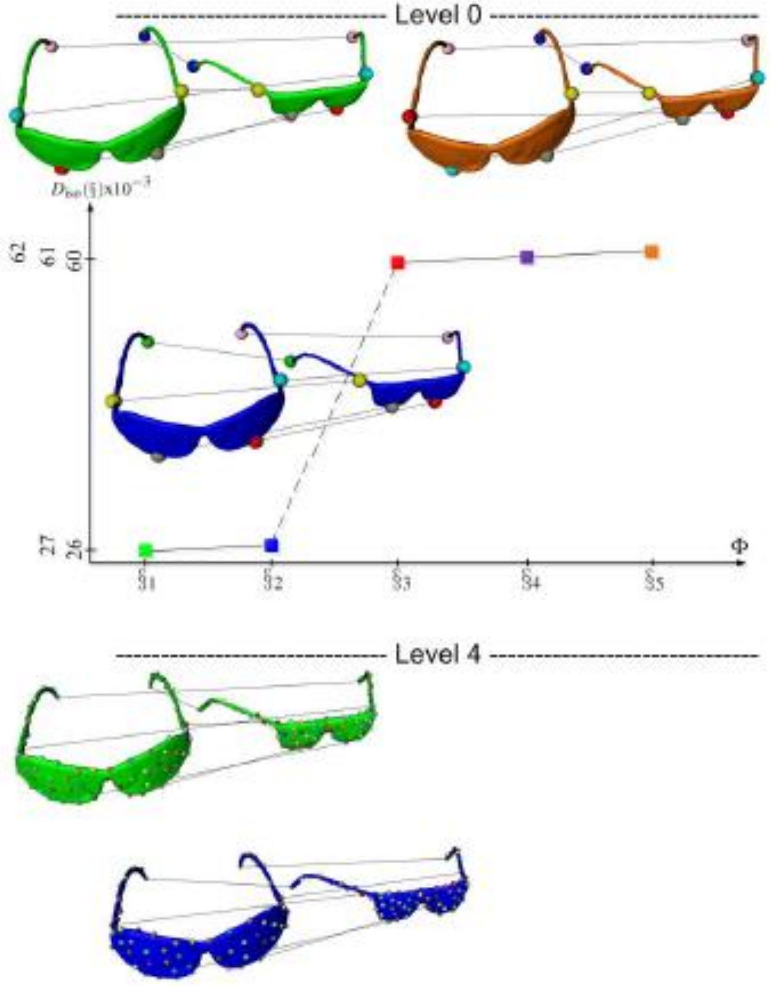


Yusuf Sahillioğlu and Yücel Yemez, Fast Dense Correspondence for Isometric Shapes, *Computer Graphics Forum (CGF)*, in revision cycle.

C2F Combinatorial Optim. w/ Tracking

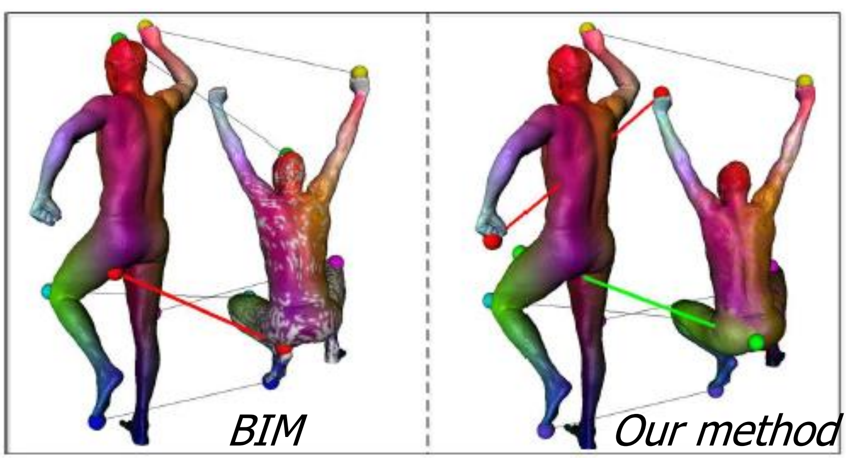
- ✓ Maps to be tracked are before the first jump in plot of initial distortions.

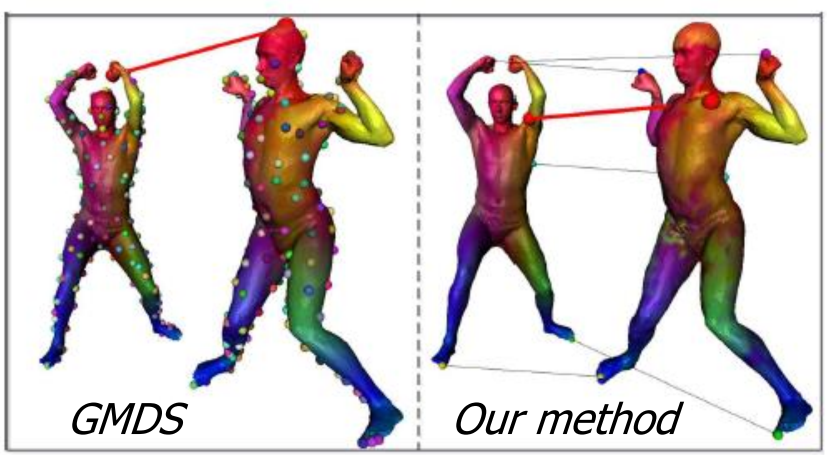




C2F Combinatorial Optim. w/ Tracking

- ✓ In addition to addressing the symmetric problem inherent to all multiresolution isometric shape matchers, this extension is tested with
  - ✓ Five benchmarks (TOSCA, Watertight, SHREC'11, SCAPE, Nonrigid World), and two state-of-the-arts (Blended Intrinsic Maps, GMDS).
  - ✓ Tracking is embedded in our C2F algorithm (SGP) as well as in GMDS.
  - ✓ Roughly speaking, 50% improvement on symmetric flips (see paper).
  - ✓ Final dense maps are better than or on a par with competitors regarding isometric distortions (see paper).





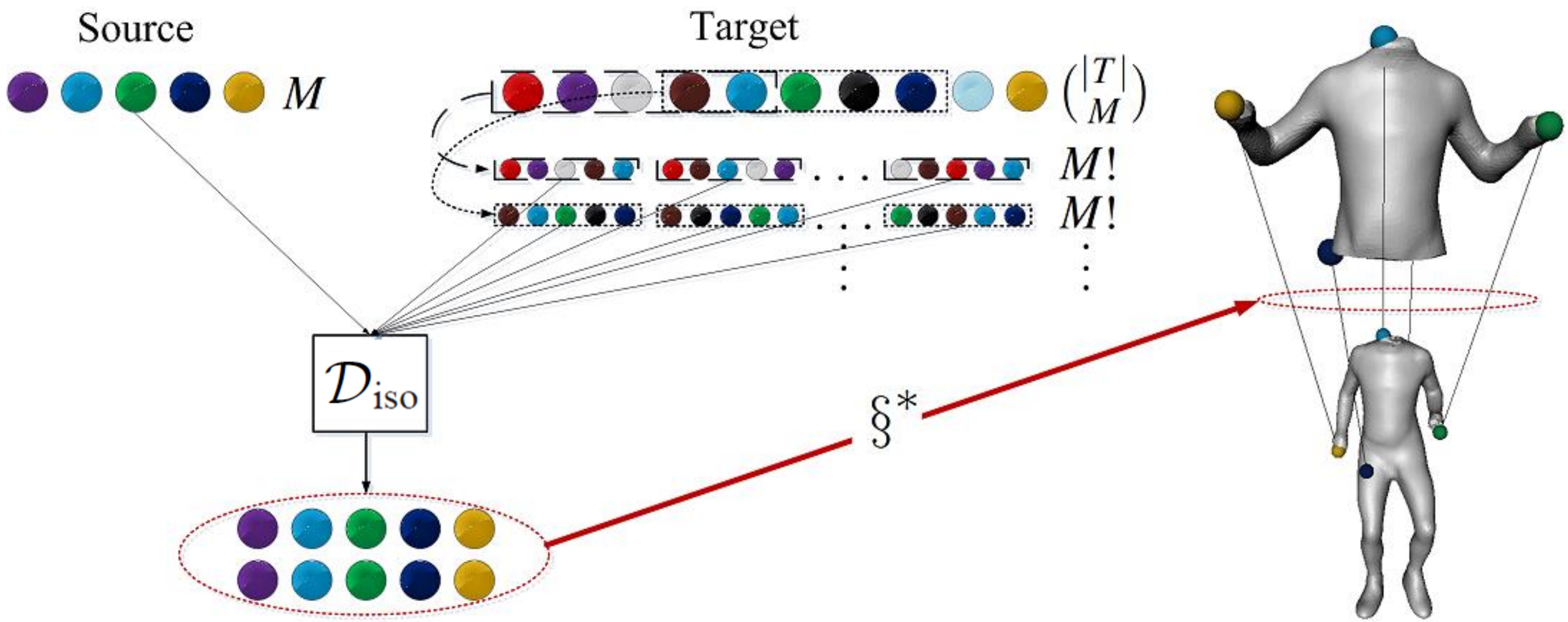
Complete Correspondence Done

- ✓ Complete shape correspondence at coarse (CVPR, PAMI) and dense (SGP) resolutions with special care on symmetric flip (CGF) for the latter is done.

- ✓ Time to match partially similar shape pairs.
  - ✓ Algorithms naturally apply to complete matching.

Combinatorial Optim. for Part Matching

- ✓ The most extreme M source vertices are matched w/ $|T|$ target extremities in the guidance of an isometric distortion measure.

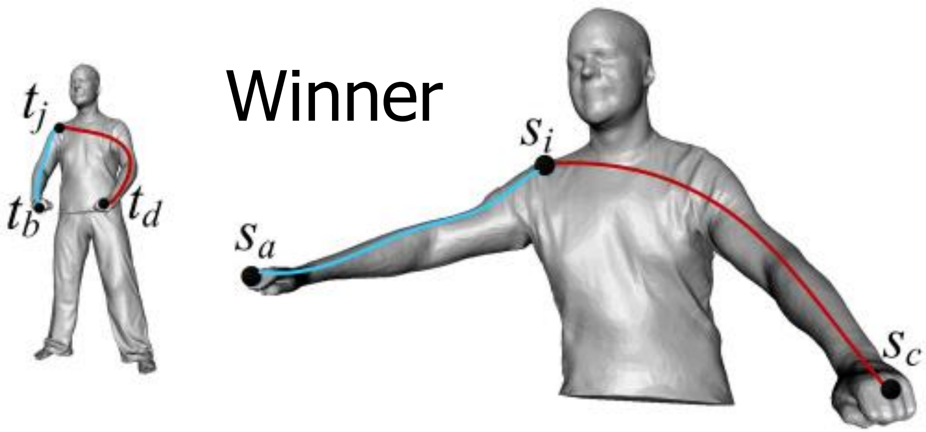


- ✓ $O(\binom{|T|}{M} M! M^3)$ computational complexity where we set M=5 in the tests.

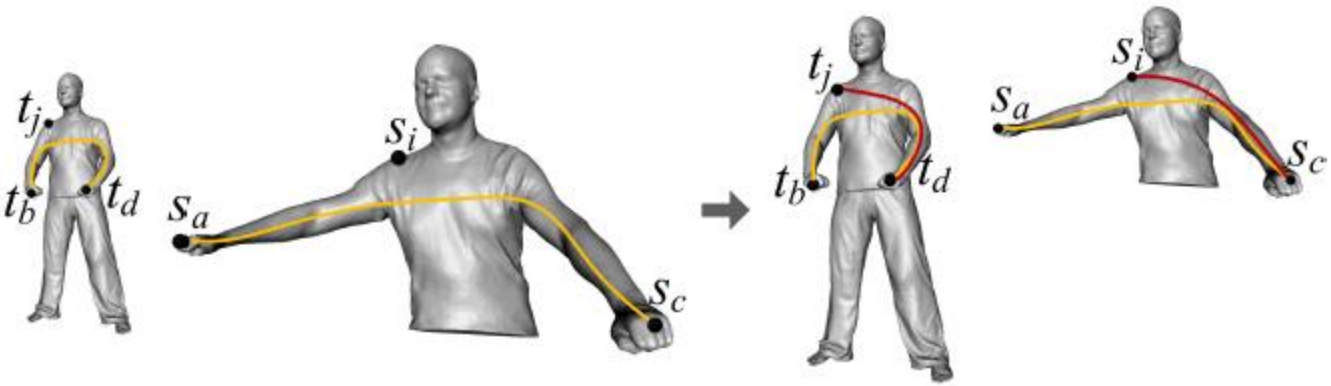
Yusuf Sahillioğlu and Yücel Yemez, Scale Normalization for Isometric Shape Matching, *Computer Graphics Forum (PG)*, submitted.

Combinatorial Optim. for Part Matching

- ✓ Two isometric distortion measures in action.
  - ✓ Scale-invariant isometric distortion $\mathcal{D}(\S)$.

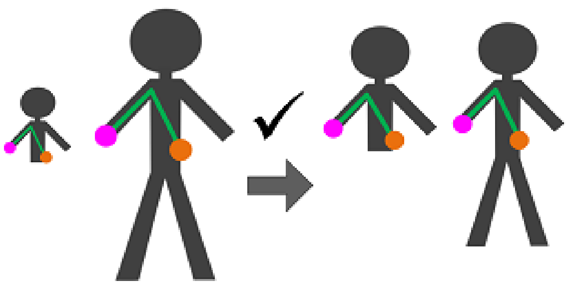


  - ✓ Isometric distortion w/ normalized geodesics.



Combinatorial Optim. for Part Matching

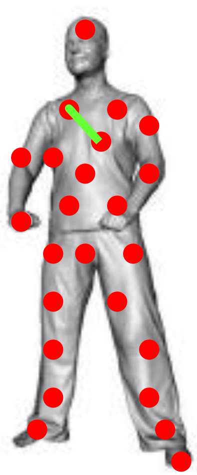
✓ Use initial coarse correspondence $\S^*$ to bring the meshes to the same scale.

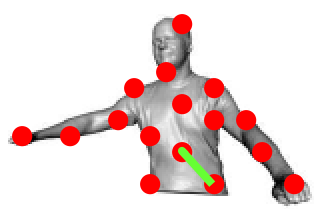


✓ Scale the target mesh by

$$\kappa = \frac{1}{\binom{|\S^*|}{2}} \sum_{((s_a,t_b),(s_c,t_d)) \in \mathcal{C}(\S^*)} \frac{g(s_a,s_c)}{g(t_b,t_d)}$$

✓ Dense sampling.



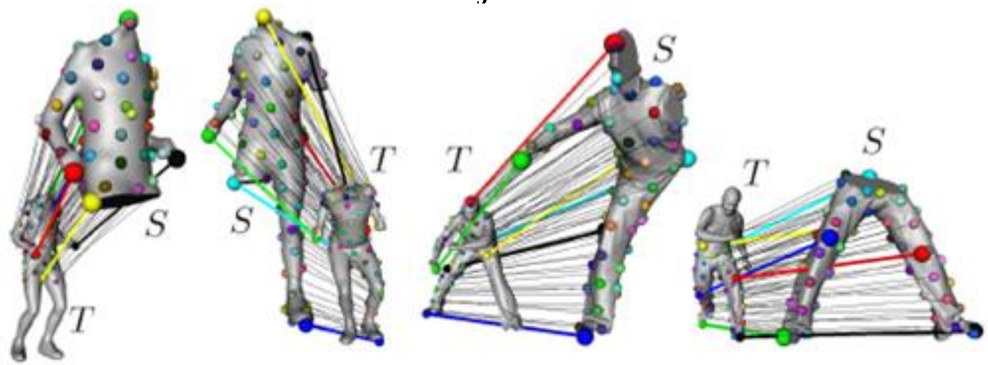


Same radius

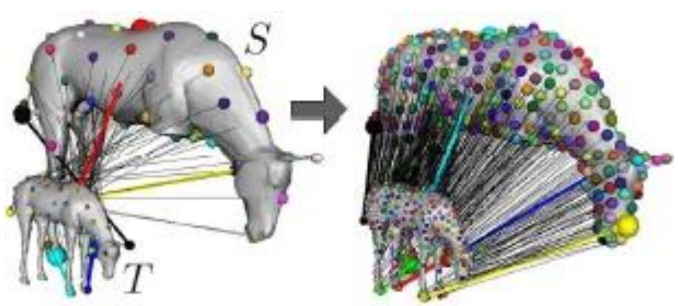
100 here

Combinatorial Optim. for Part Matching

- ✓ Dense matching.
  - ✓ Minimum-weight perfect matching on cost matrix $\mathbf{C}$.
    - ✓ $c_{i,j}$ = cost of matching s_i to t_j //generating $\S^*$ is traversed by (s_i, t_j).

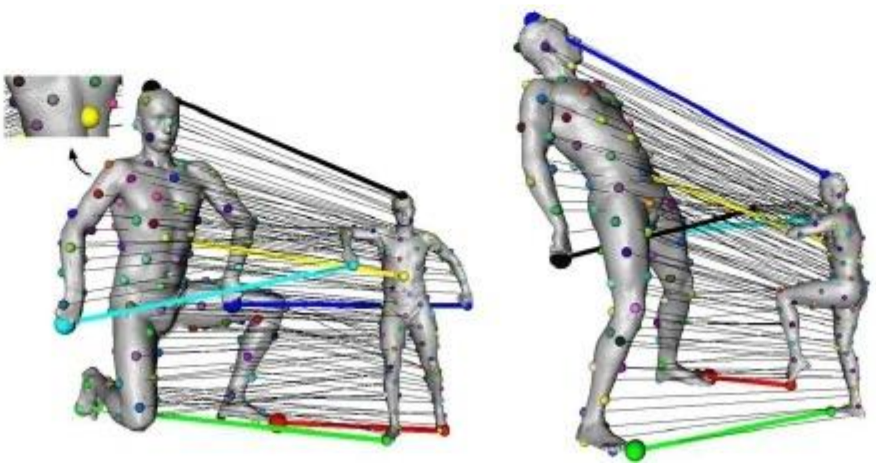


- ✓ Symmetric flip caring: repeat above (scaling, sampling, matching) with K-1 more generating initial coarse correspondences that follow $\S^*$ in sorted distortions list.

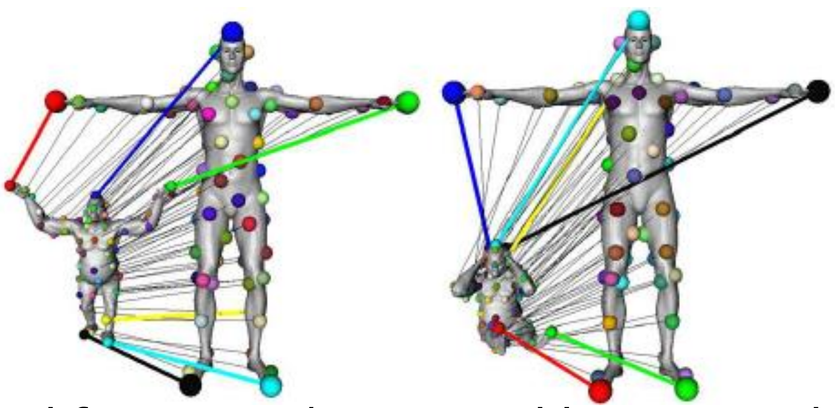


Combinatorial Opt. for Part Matching (Results)

- ✓ Not only for part matching

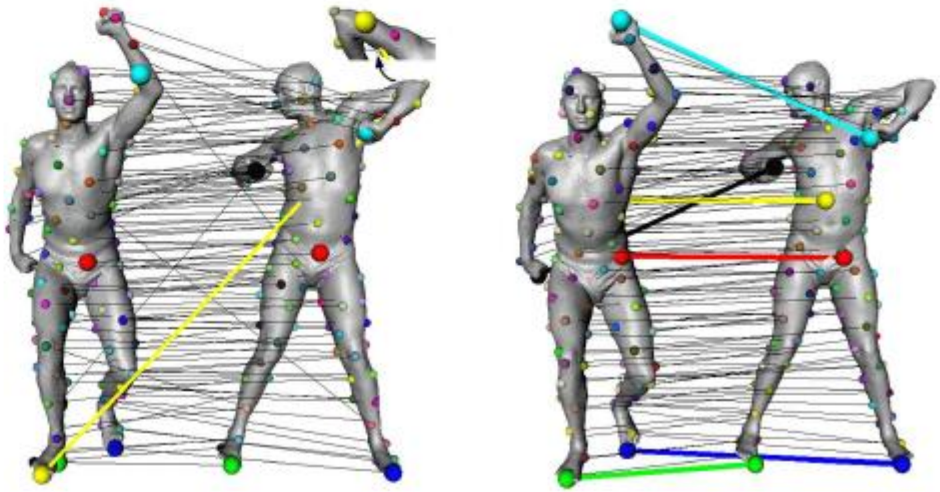


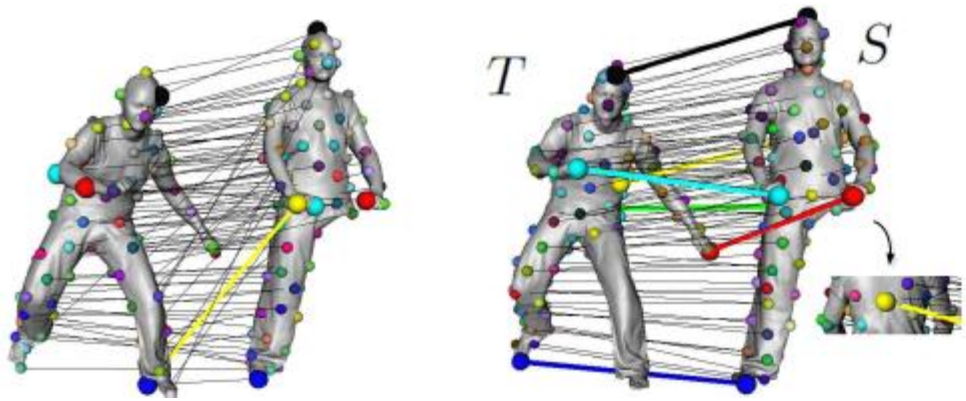
but also for complete matching



and for pairs w/ incompatible max geodesics.

- ✓ Comparison w/ Möbius Voting (MV).





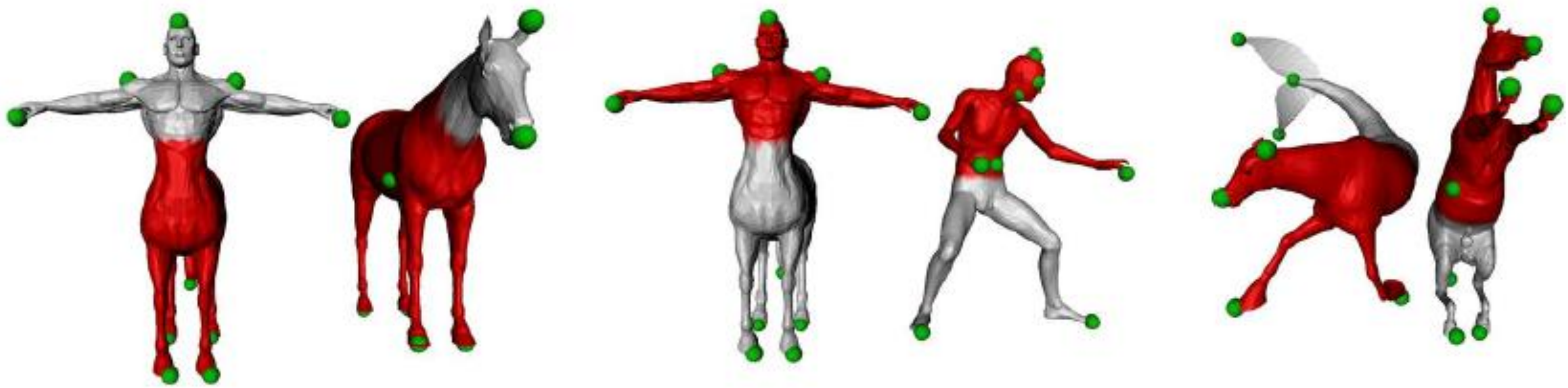
MV: bad extremity matching, triangulation.

Combinatorial Opt. for Part Matching (Limitation)

- Presence of uncommon parts may fail this framework which forces to match $M=5$ most extremes as a whole.
  - Embedding $\mathcal{D}_{\mathrm{iso}}$ into a more sophisticated framework should help as it handles arbitrary scaling of the similar parts.
  - Solution is our rank-and-vote-and-combine (RAVAC) algorithm.

RAVAC Optimization

- ✓ Multiple common parts at arbitrary scales as well as uncommon parts.



- ✓ Find sparse correspondence b/w shape extremes (green spheres) which will then be extended to a denser one.

- ✓ Handles shape pairs w/ small similarity overlap (red regions), the smallest indeed to the best of our knowledge.

RAVAC Optimization

- ✓ Ranking
  - ✓ Explore the space of all possible partial maps b/w shape extremities to rank them w.r.t. the isometric distortion $\tilde{d}_{\text{iso}}$ they yield.

$$d_{\text{iso}}(s_i, t_j, \S') = \frac{1}{|\S'|} \sum_{(s_l, t_m) \in \S'} |g(s_i, s_l) - g(t_j, t_m)|$$

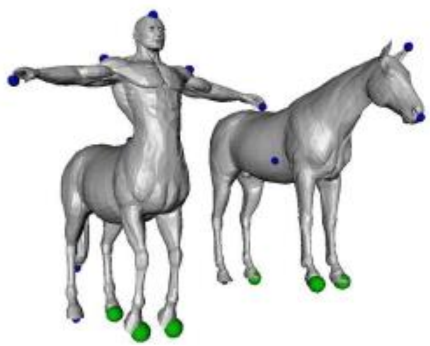
$$\tilde{d}_{\text{iso}}(s_i, t_j) = \frac{1}{4} \sum_{k \in [2,5]} \min_l \{ d_{\text{iso}}(s_i, t_j, \S_l^{(k)}) \}$$

$\{\S_l^{(k)} \mid l = 1, 2, \ldots, L_k\}$: set of all maps of size k, not including (s_i, t_j).

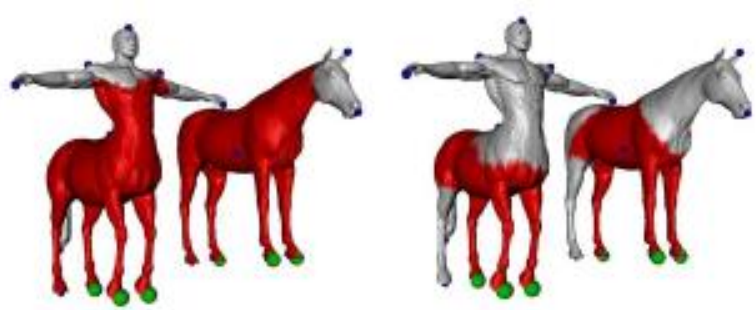
  - ✓ Qualify matches w/ relatively low distortions, i.e., top-ranked.

RAVAC Optimization

- ✓ Voting
  - ✓ Qualified top-ranked matches analyzed at a denser reso to obtain confidences.
  - ✓ For each triplet of samples from source & target (potentially compatible greens)

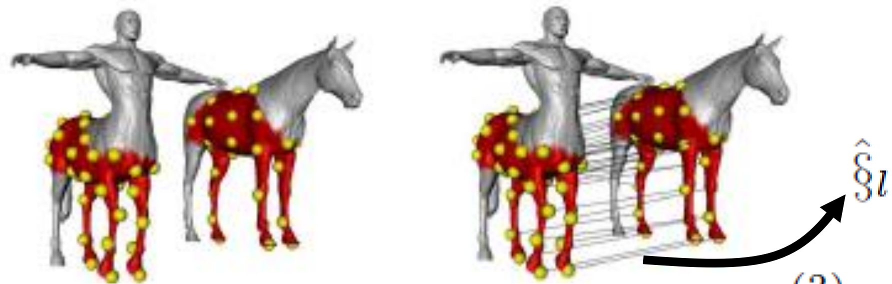


  - ✓ Generate a safe map $\S_l^{(3)} = \{(s_{i_1}, t_{j1}), (s_{i_2}, t_{j2}), (s_{i_3}, t_{j3})\}$ where all pairs are qualified.
  - ✓ Bring meshes to the same scale via $\kappa = (\frac{g(s_{i_1}, s_{i_2})}{g(t_{j_1}, t_{j_2})} + \frac{g(s_{i_1}, s_{i_3})}{g(t_{j_1}, t_{j_3})} + \frac{g(s_{i_2}, s_{i_3})}{g(t_{j_2}, t_{j_3})})/3$.
  - ✓ Decide the regions of interests.

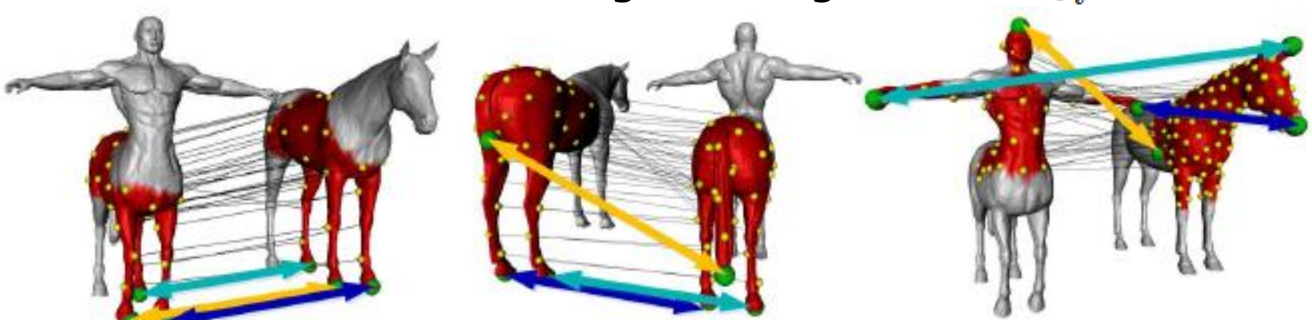


RAVAC Optimization

- ✓ Spread and match evenly-spaced dense samples on regions.



- ✓ Add confidence votes to the generating matches $\S_l^{(3)}$ that accumulate in $\mathbf{\Gamma}$

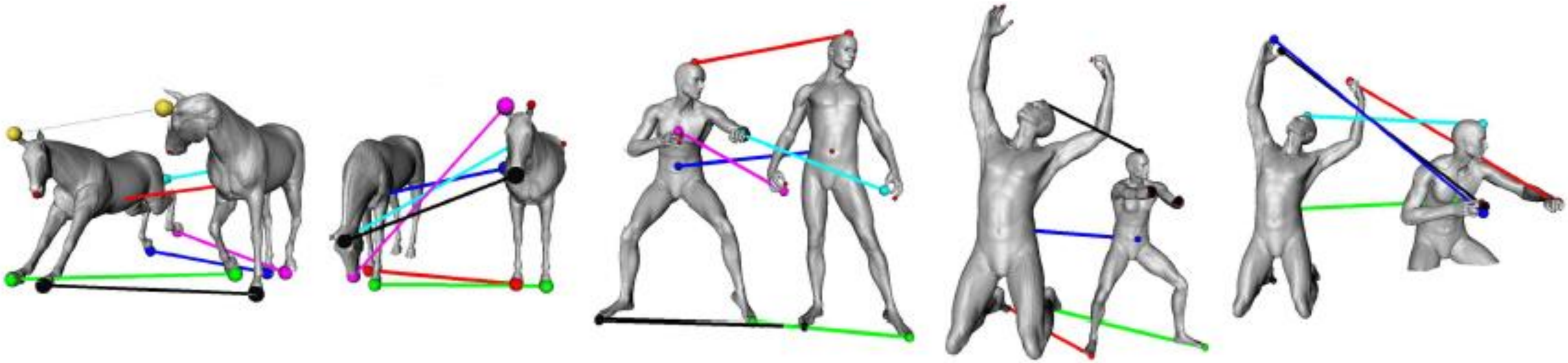


via $\gamma_{ij} = \gamma_{ij} + \exp(-d_{\text{iso}}(s_i, t_j, \hat{\S}_l))$ where $(s_i, t_j) \in \S_l^{(3)}$.

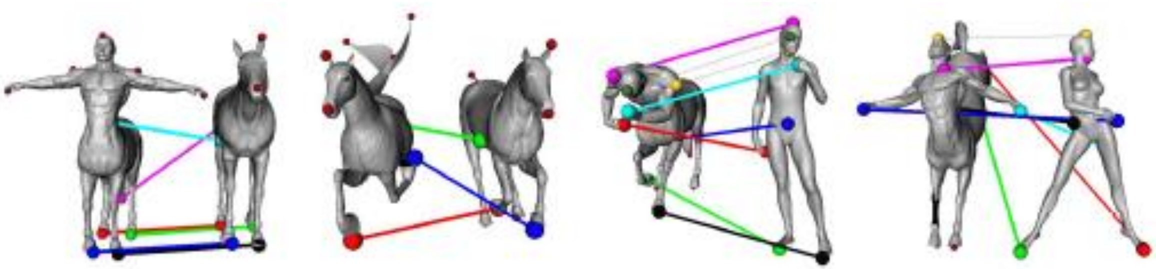
- ✓ Yet another example w/ a different generating pair of sample triplets.

RAVAC Optimization

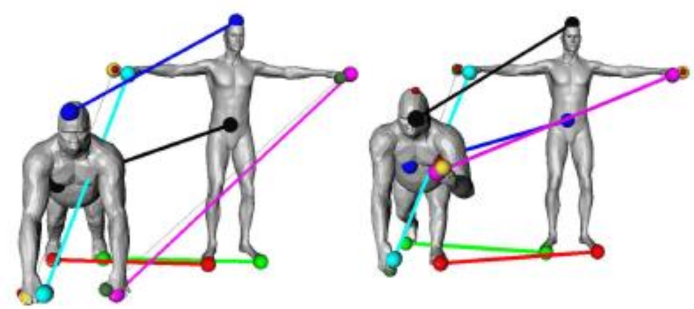
- ✓ Combining
  - ✓ Iterate bipartite graph matching based on vote matrix Γ by removing the least confident match at the end of each iteration.
- ✓ Complete correspondence and part matching are handled naturally.



- ✓ The harder case with uncommon parts.

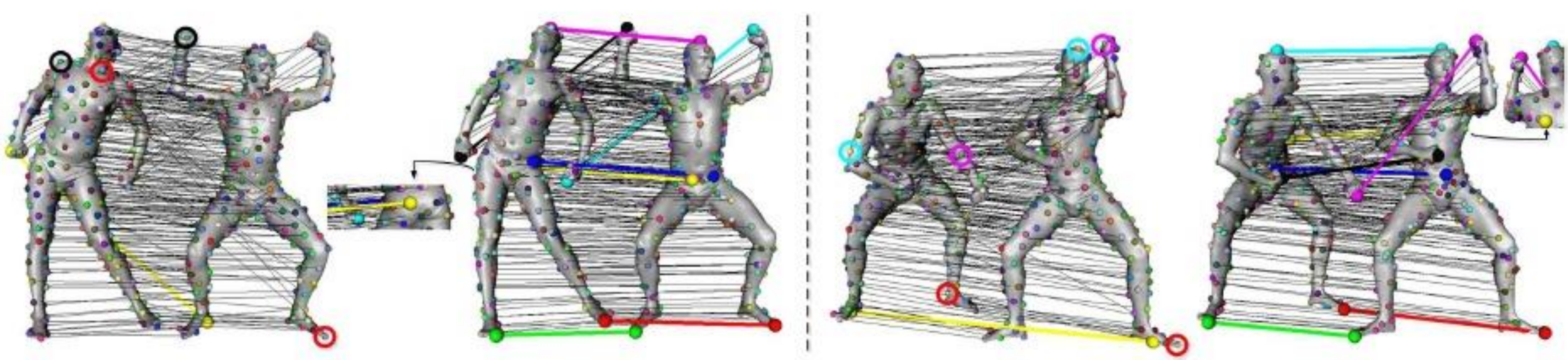


- ✓ Locally similar, globally not.



RAVAC Optimization

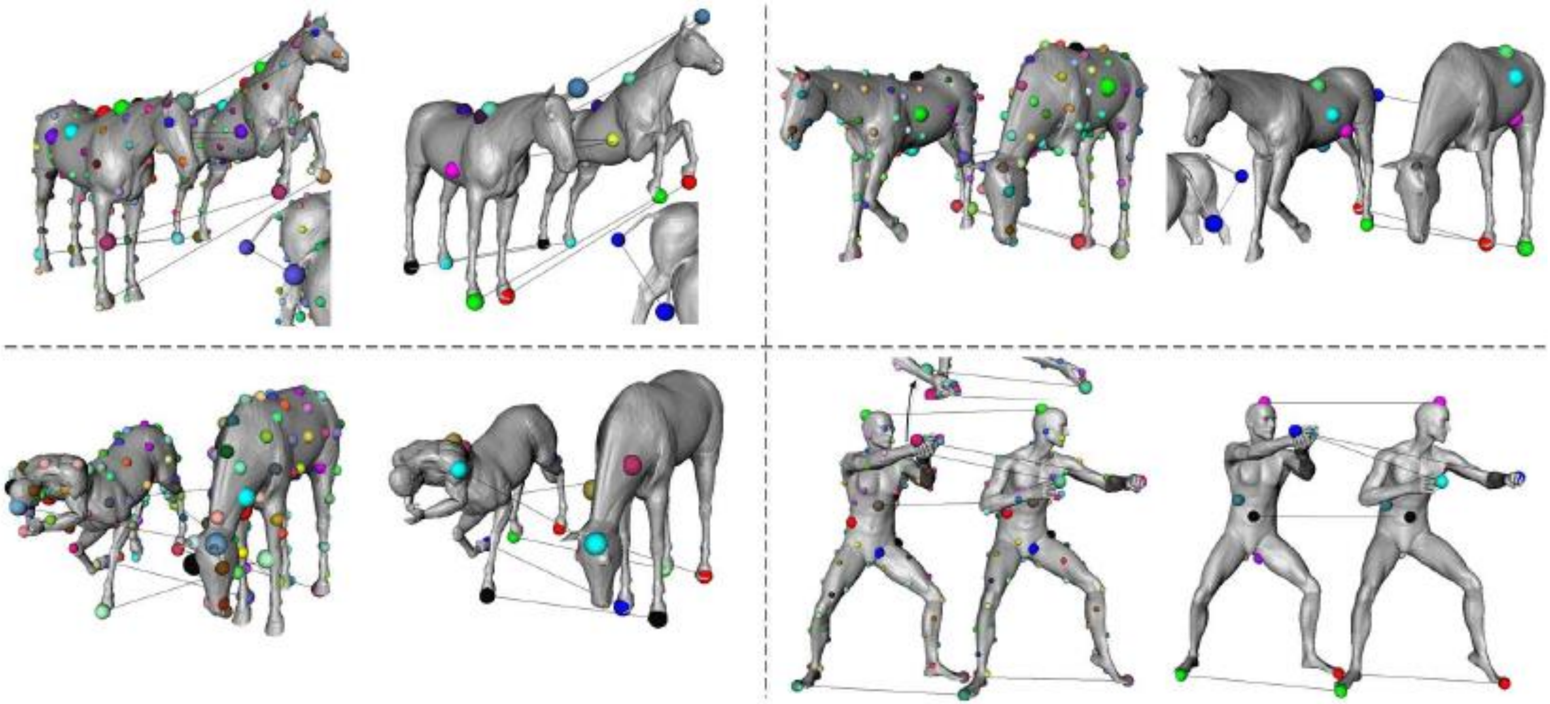
- ✓ Extension to dense map
  - ✓ For each map of size 3 chosen from optimal coarse correspondence, densely sample and match the regions as before (overlap trick in sampling).
  - ✓ $F(\hat{s}_i)$: set of matches for dense source sample $\hat{s}_i$.
  - ✓ Geodesic centroid of $F(\hat{s}_i)$ is then $\mathbf{b}_i = \frac{1}{|F(\hat{s}_i)|} \sum_{\hat{t}_j \in F(\hat{s}_i)} \hat{\mathbf{t}}_j$ which gives the dense match $(\hat{s}_i, \hat{t}_k)$ where $\hat{t}_k$ is a target vertex closest to $\mathbf{b}_i$.



In comparison w/ Möbius Voting (1st and 3rd pairs from the left).

RAVAC Optimization (Comparisons & Limitations)

- ✓ More Möbius Voting comparisons.



- ✓ Limitations
  - ✓ Each part to be matched must be represented by at least 3 samples, which is generally the case anyway.
  - ✓ Incorporate diffusion-based metrics for topological noise robustness.

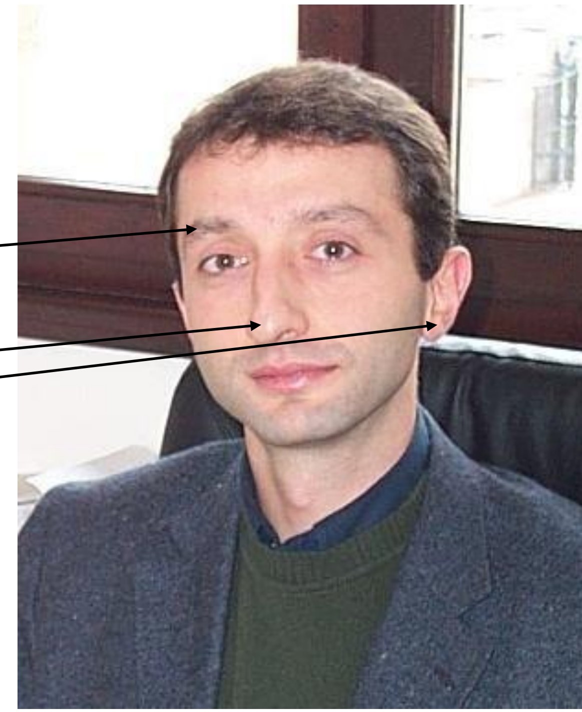
Conclusions & Future Work

- ✓ Four new sampling algorithms.
- ✓ Isometric distortion functions and their optimizers in 3D Euclidean space.
- ✓ The fastest computational complexity on dense correspondence.
- ✓ Symmetric flip handling for all multiresolution isometric shape matchers.
- ✓ Partial correspondence for shapes w/ significantly small similarity overlap.
- ✓ Correspondences that are partial and dense at the same time.
- ✓ Insensitivity to shape topology and peculiarities of the triangulation.

- ✓ Investigate tradeoff b/w the accuracy of the geodesic metric in use and topological noise robustness of the diffusion-based metrics to be tested.
- ✓ Incorporate more shapes into the process to establish or improve correspondences (Done during post-doc: [PG'14]).

People

Yusuf,
PhD student



Assoc. Prof. Yücel Yemez,
supervisor