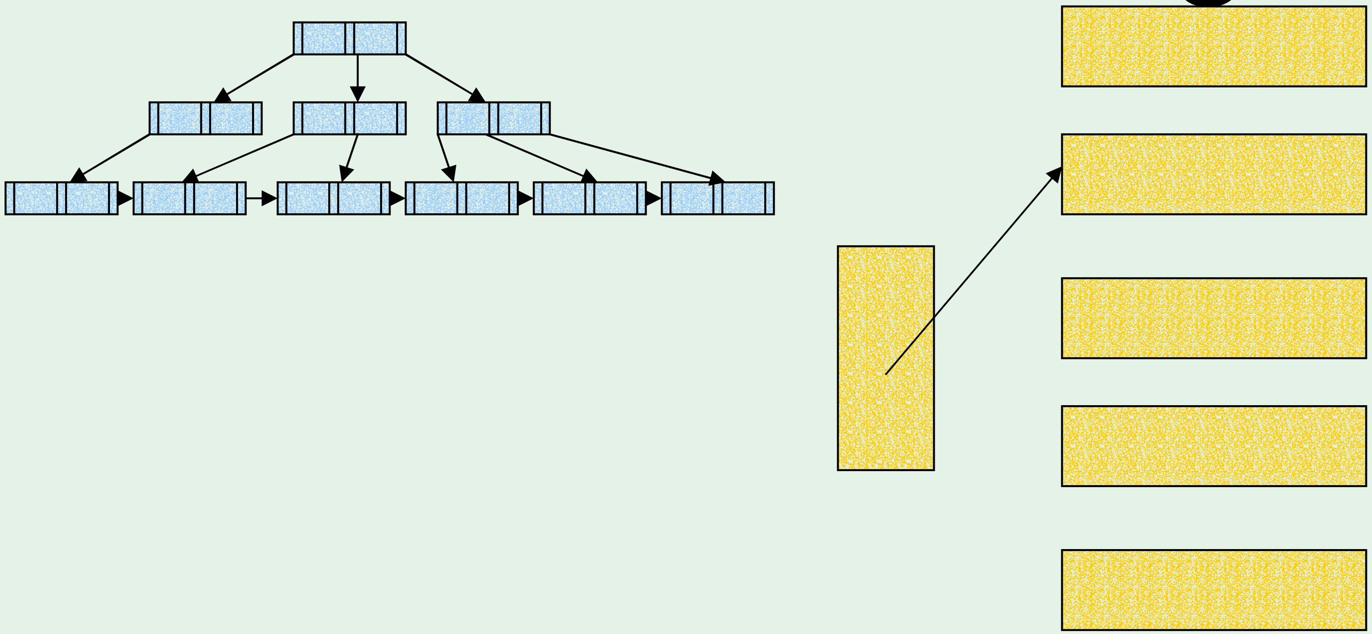
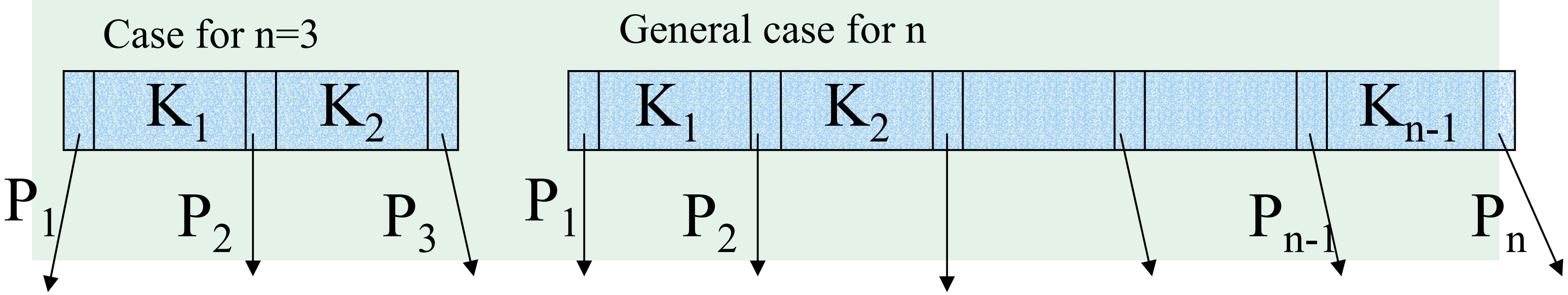
B+ Tree and Hashing



- B+ Tree Properties
- B+ Tree Searching
- B+ Tree Insertion
- B+ Tree Deletion
- Static Hashing
- Extendable Hashing
- Questions in pass papers

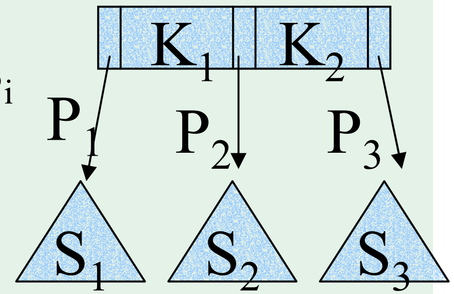
- Balanced Tree
  - Same height for paths from root to leaf
  - Given a search-key K, nearly same access time for different K values
- B+ Tree is constructed by parameter **n**
  - Each Node (except root) has ⌈n/2⌉ to n pointers
  - Each Node (except root) has ⌈n/2⌉-1 to n-1 search-key values



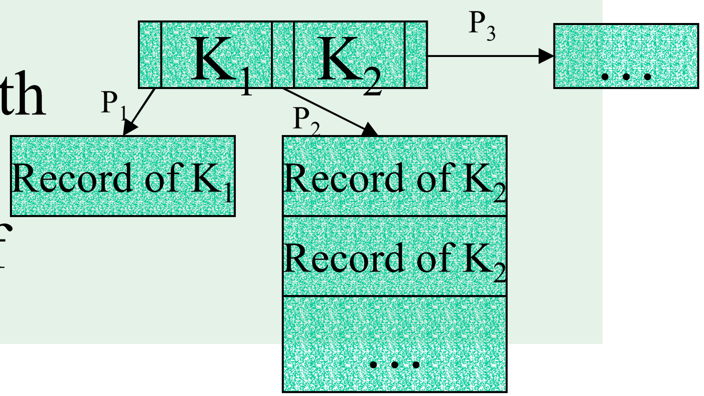
- Search keys are sorted in order
  - $K_1 < K_2 < \ldots < K_{n-1}$
- Non-leaf Node
  - Each key-search values in subtree S_i pointed by $P_i < K_i$, $>= K_{i-1}$

    Key values in $S_1 < K_1$

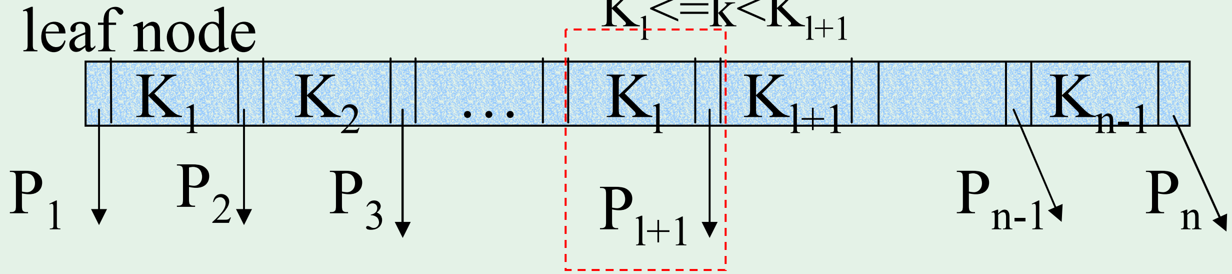
    $K_1 <=$ Key values in $S_2 < K_2$



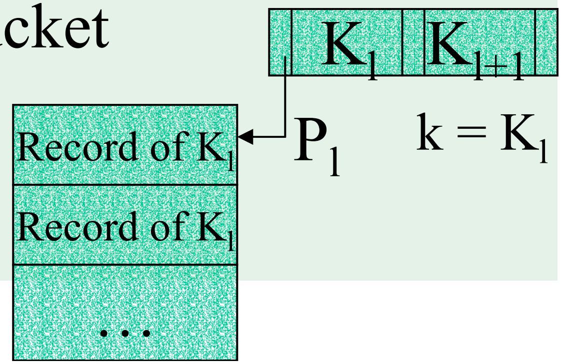
- Leaf Node
  - P_i points record or bucket with search key value K_i
  - P_n points to the neighbor leaf node



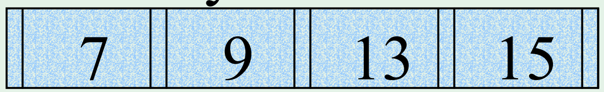
- Given a search-value k
  - Start from the root, look for the **largest** search-key value (K_l) in the node <= k
  - Follow pointer P_{l+1} to next level, until reach a leaf node



  - If k is found to be equal to K_l in the leaf, follow P_l to search the record or bucket



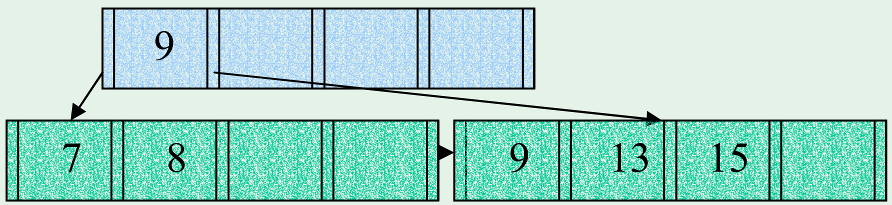
- Overflow
  - When number of search-key values exceed n-1



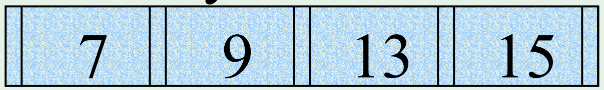
Insert 8

– Leaf Node

- Split into two nodes:
  - 1st node contains $\lceil (n-1)/2 \rceil$ values
  - 2nd node contains remaining values
  - Copy the smallest search-key value of the 2nd node to parent node



- Overflow
  - When number of search-key values exceed n-1



Insert 8

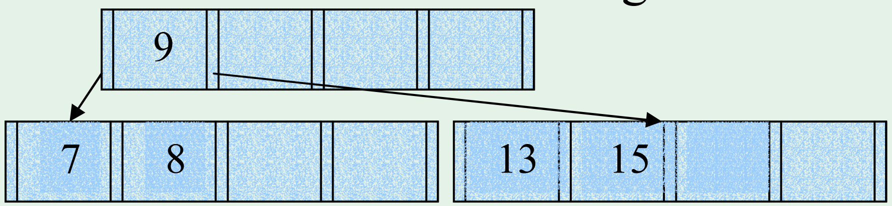
–Non-Leaf Node

•Split into two nodes:

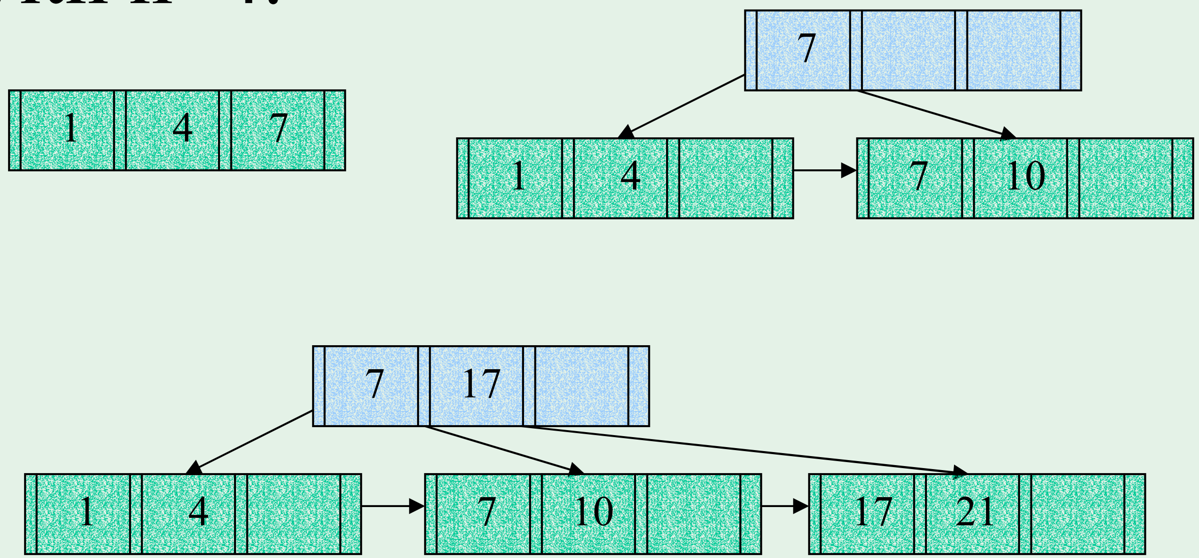
–1st node contains ⌈n/2⌉-1 values

–Move the smallest of the remaining values, together with pointer, to the parent

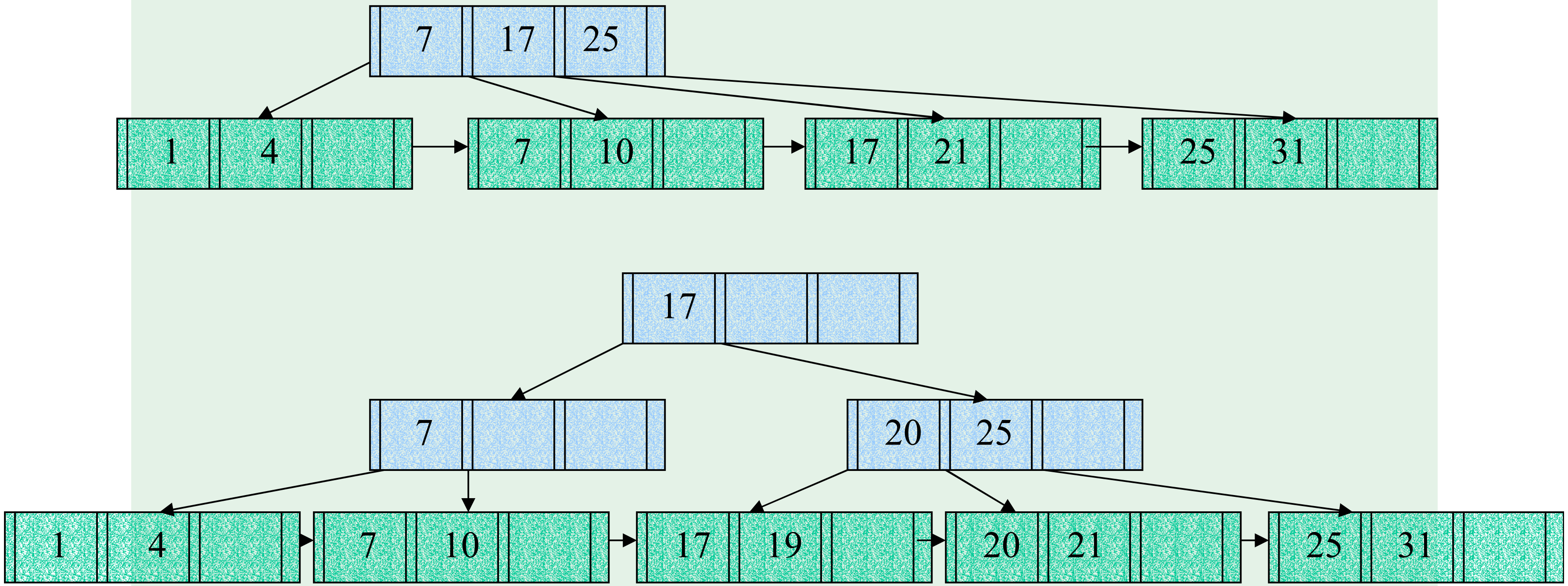
–2nd node contains the remaining values



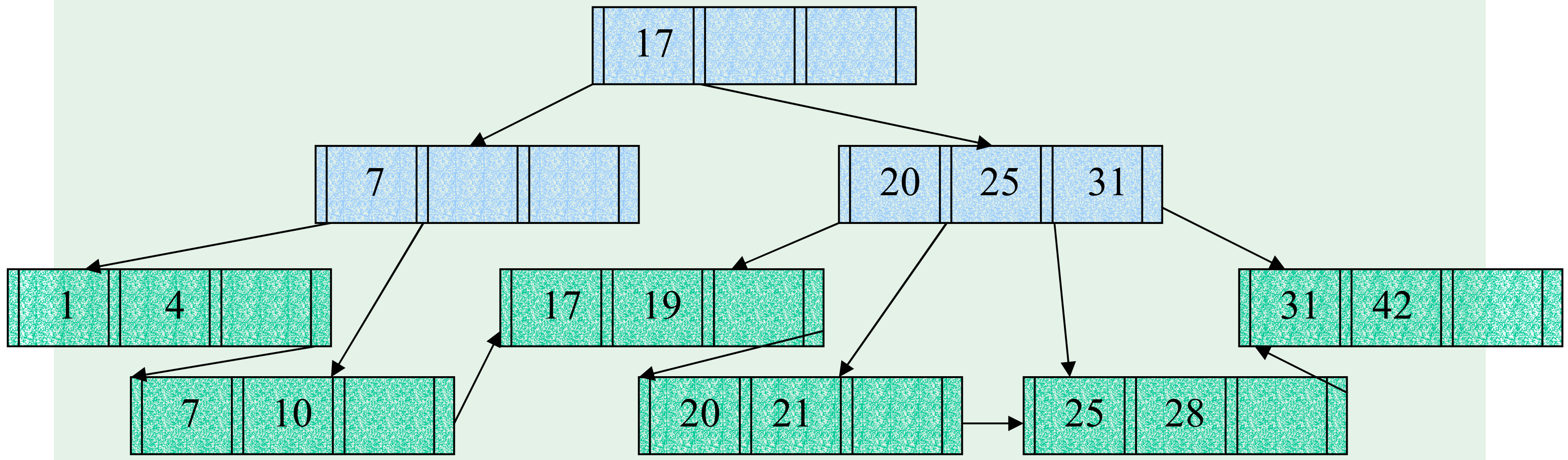
- Example 1: Construct a B$^+$ tree for (1, 4, 7, 10, 17, 21, 31, 25, 19, 20, 28, 42) with n=4.



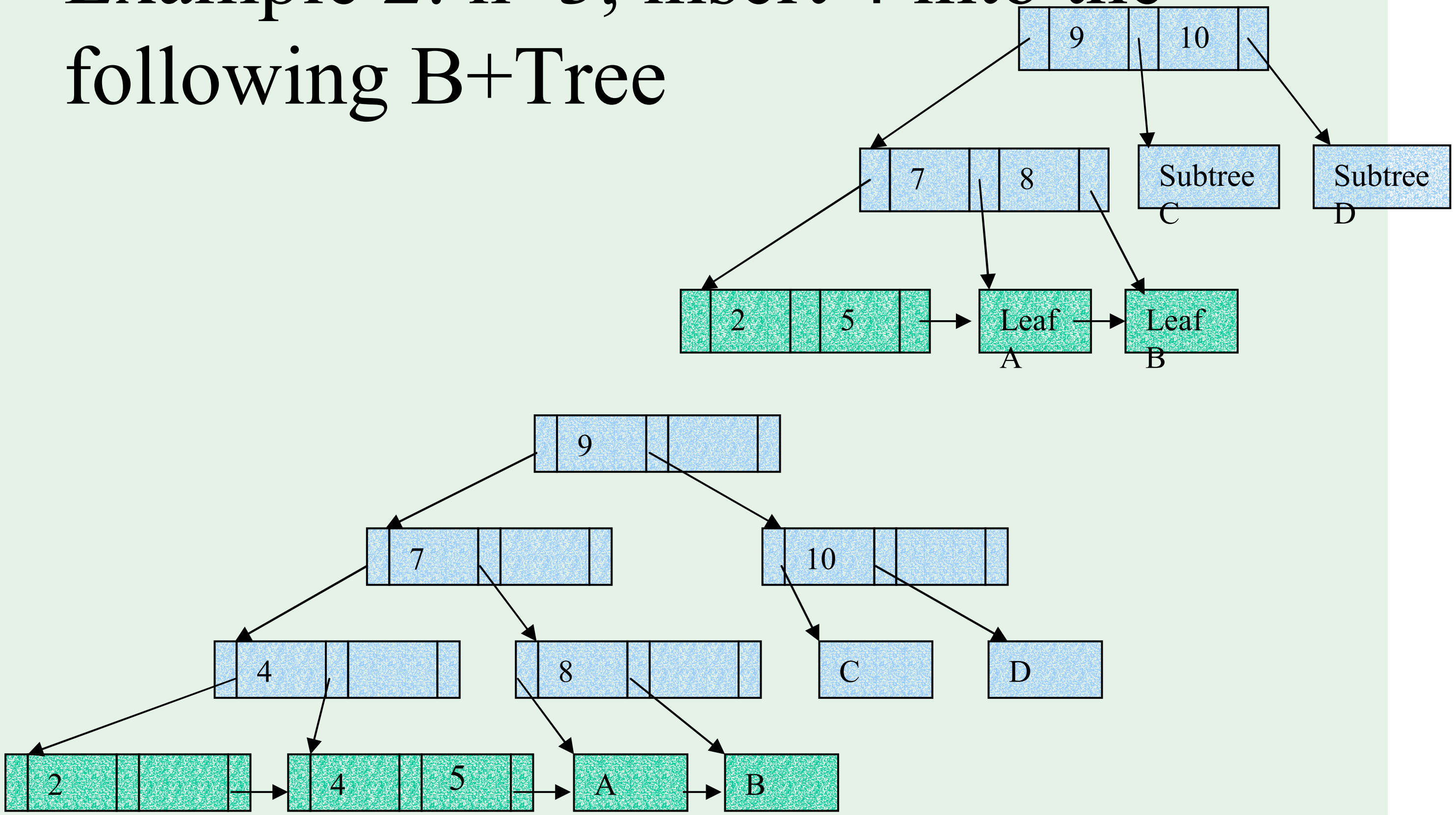
- 1, 4, 7, 10, 17, 21, 31, 25, 19, 20, 28, 42



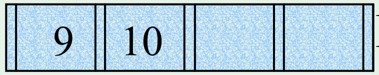
• 1, 4, 7, 10, 17, 21, 31, 25, 19, 20, 28, 42



- Example 2: n=3, insert 4 into the following B+Tree



- Underflow

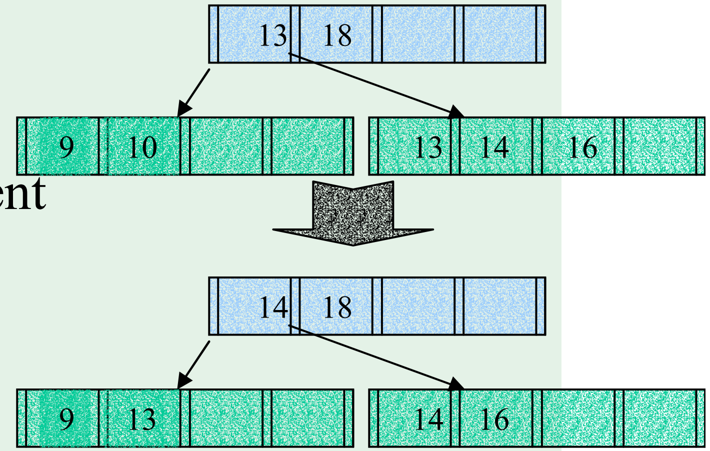


Delete 10

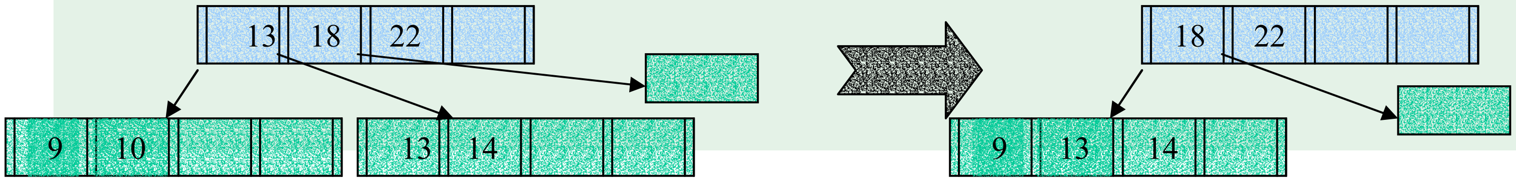
  – When number of search-key values $< \lceil n/2 \rceil - 1$

–Leaf Node

- •Redistribute to sibling
  - •Right node not less than left node
  - •Replace the between-value in parent by their smallest value of the right node

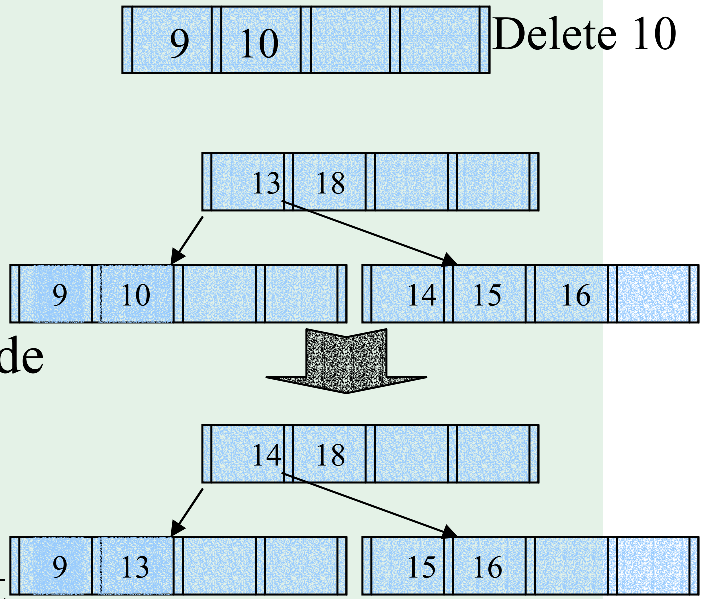


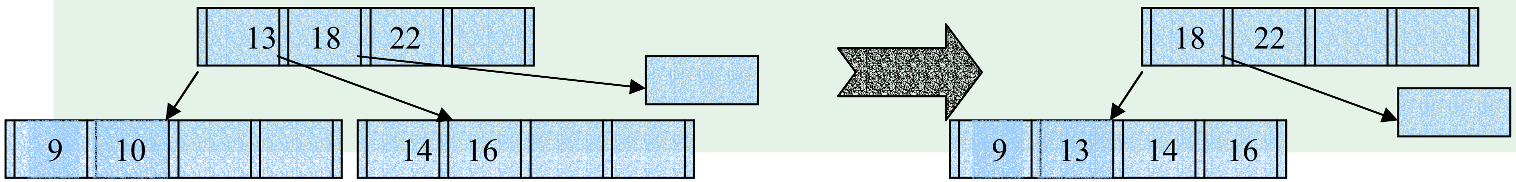
- •Merge (contain too few entries)
  - •Move all values, pointers to left node
  - •Remove the between-value in parent



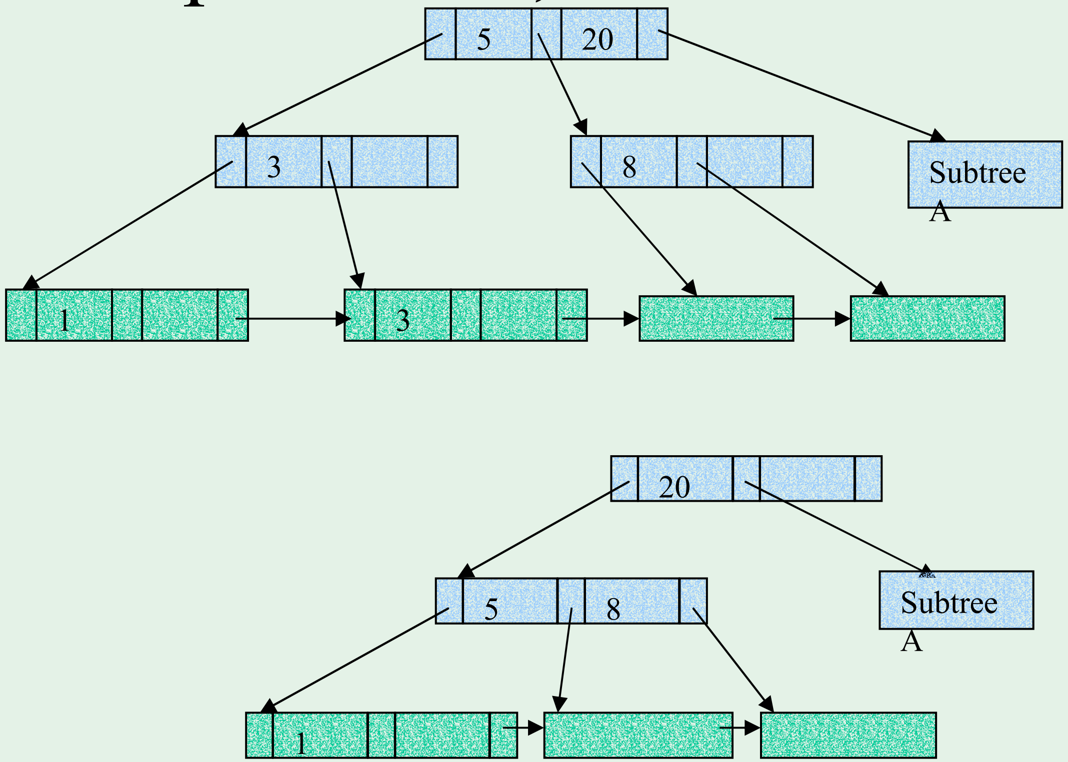
–Non-Leaf Node

- •Redistribute to sibling
  - •Through parent
  - •Right node not less than left node
- •Merge (contain too few entries)
  - •Bring down parent
  - •Move all values, pointers to left node
  - •Delete the right node, and pointers in parent

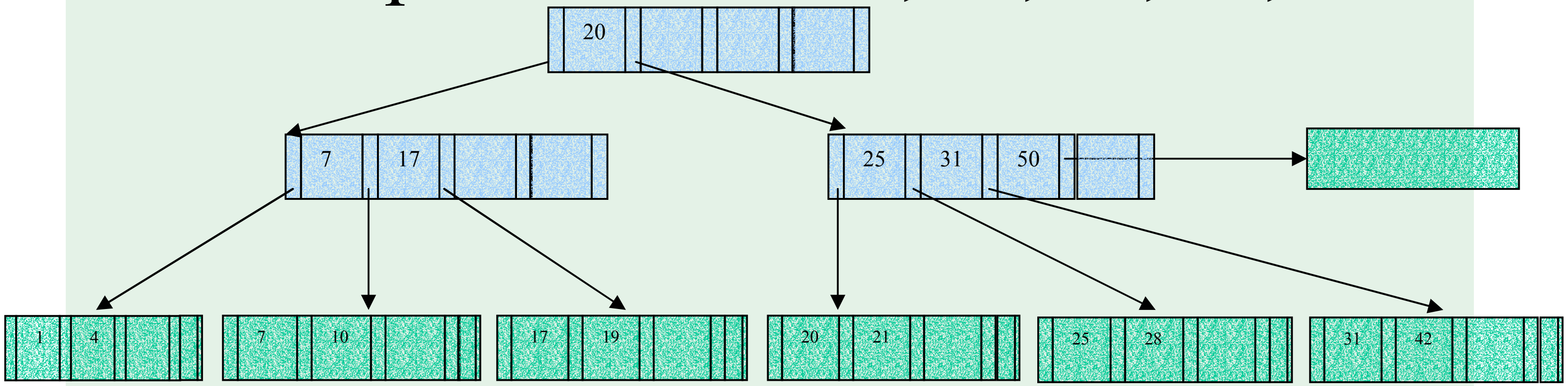


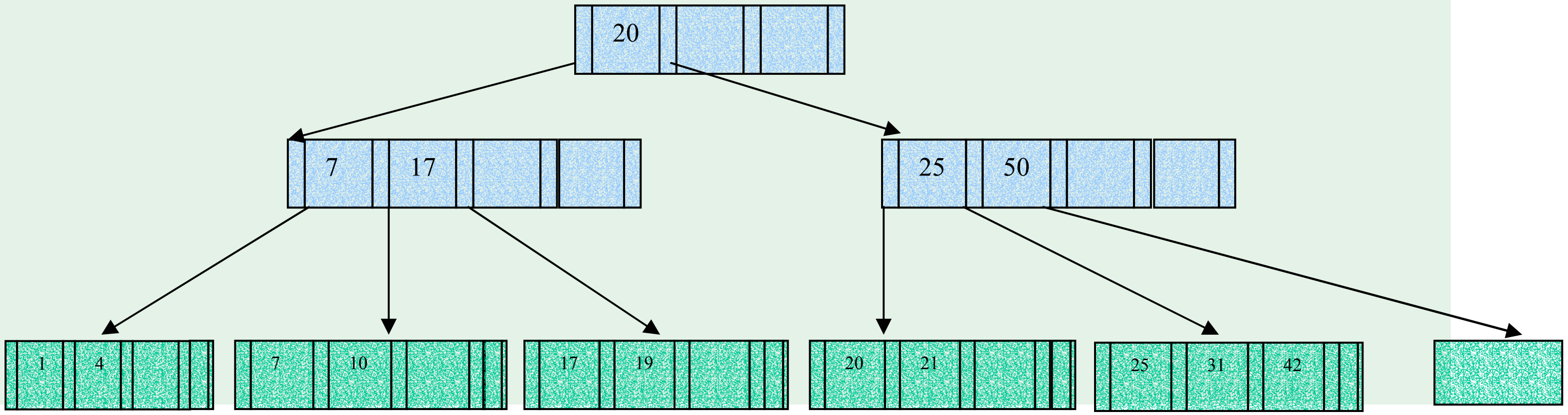


- Example 3: n=3, delete 3

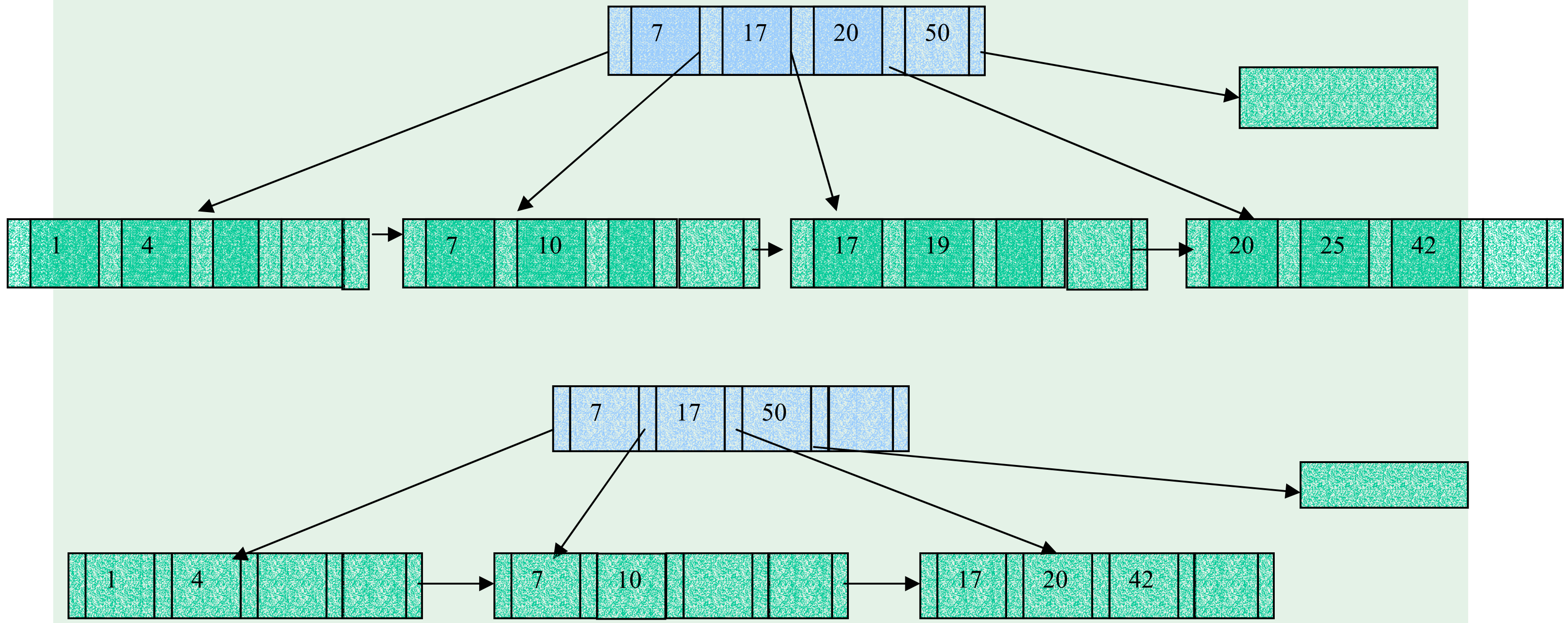


- Example 4: Delete 28, 31, 21, 25, 19

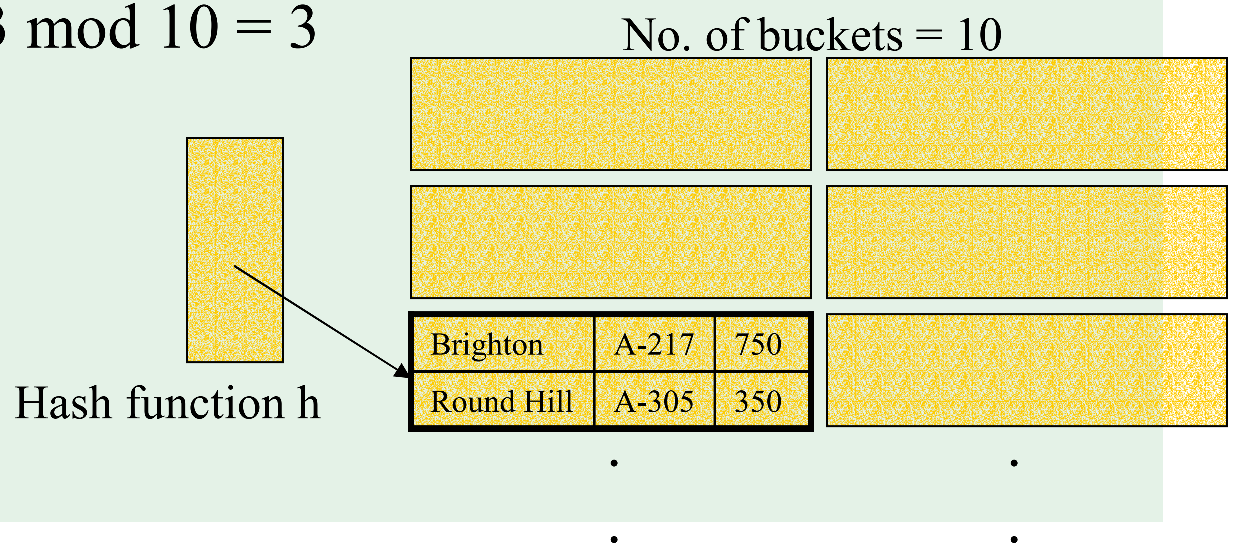


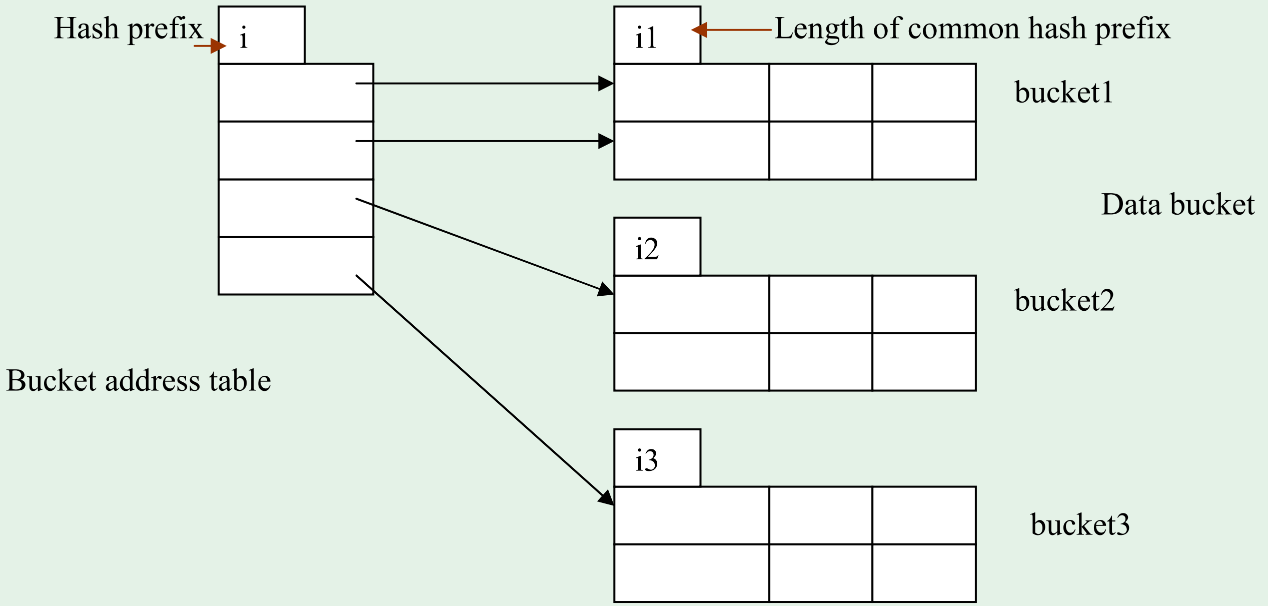


- Example 4: Delete 28, 31, 21, 25, 19



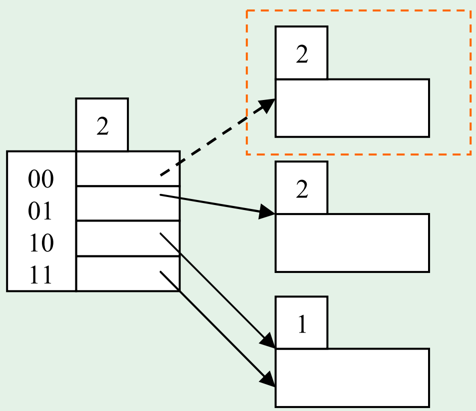
- A hash function h maps a search-key value K to an address of a bucket
- Commonly used hash function *hash value mod* n_B where n_B is the no. of buckets
- E.g. h(Brighton) = (2+18+9+7+8+20+15+14) mod 10 = 93 mod 10 = 3

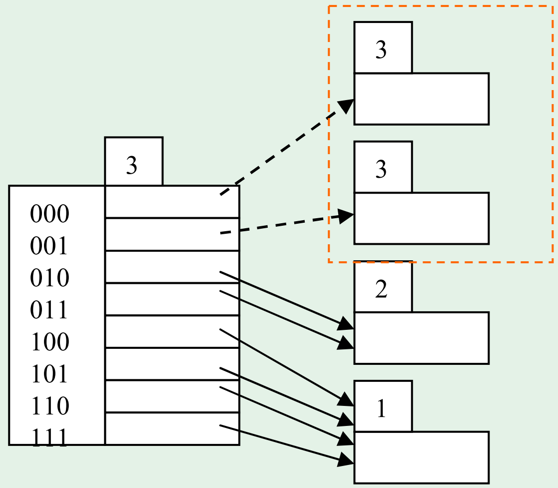




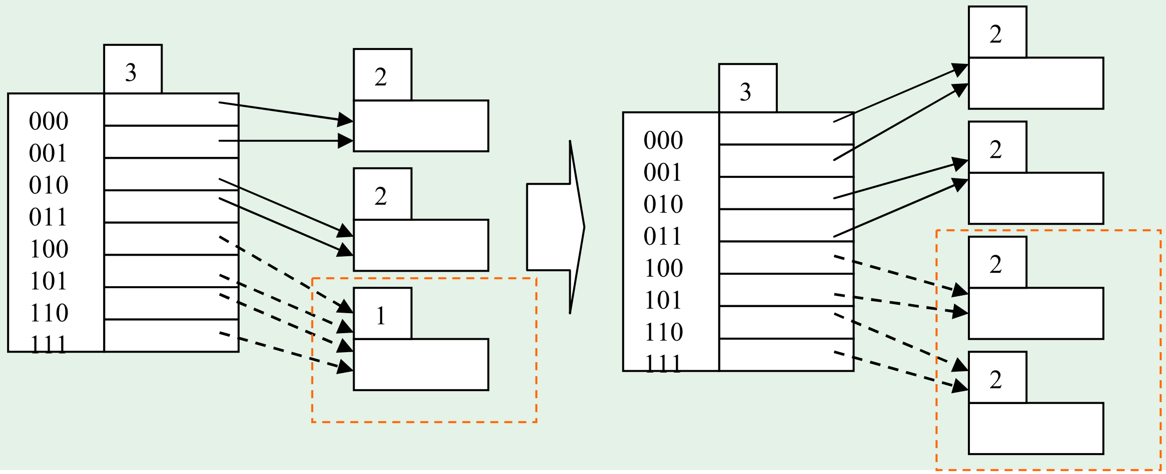
- Hash function returns **b** bits
- Only the prefix **i** bits are used to hash the item
- There are $\mathbf{2^i}$ entries in the bucket address table
- Let $\mathbf{i_j}$ be the length of the common hash prefix for data bucket **j**, there is $\mathbf{2^{(i-i_j)}}$ entries in bucket address table points to **j**

- Splitting (Case 1 i_j=i)
  - Only one entry in bucket address table points to data bucket j
  - i++; split data bucket j to j, z; i_j=i_z=i; rehash all items previously in j;



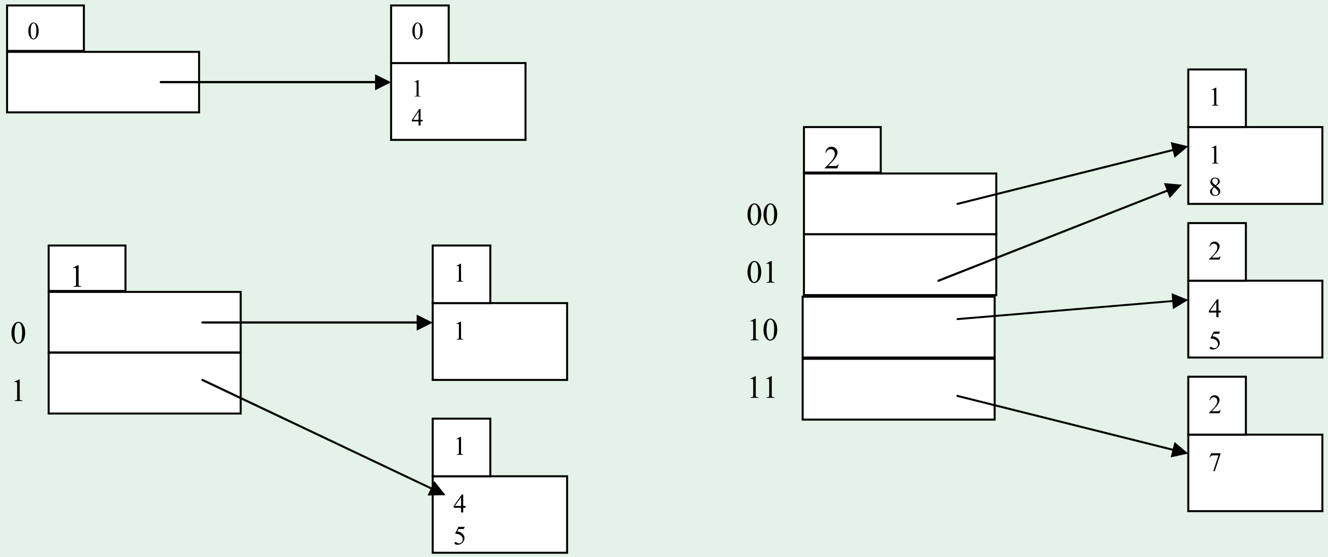


- Splitting (Case 2 $i_j < i$)
  - More than one entry in bucket address table point to data bucket j
  - split data bucket j to j, z; $i_j = i_z = i_j + 1$; Adjust the pointers previously point to j to j and z; rehash all items previously in j;



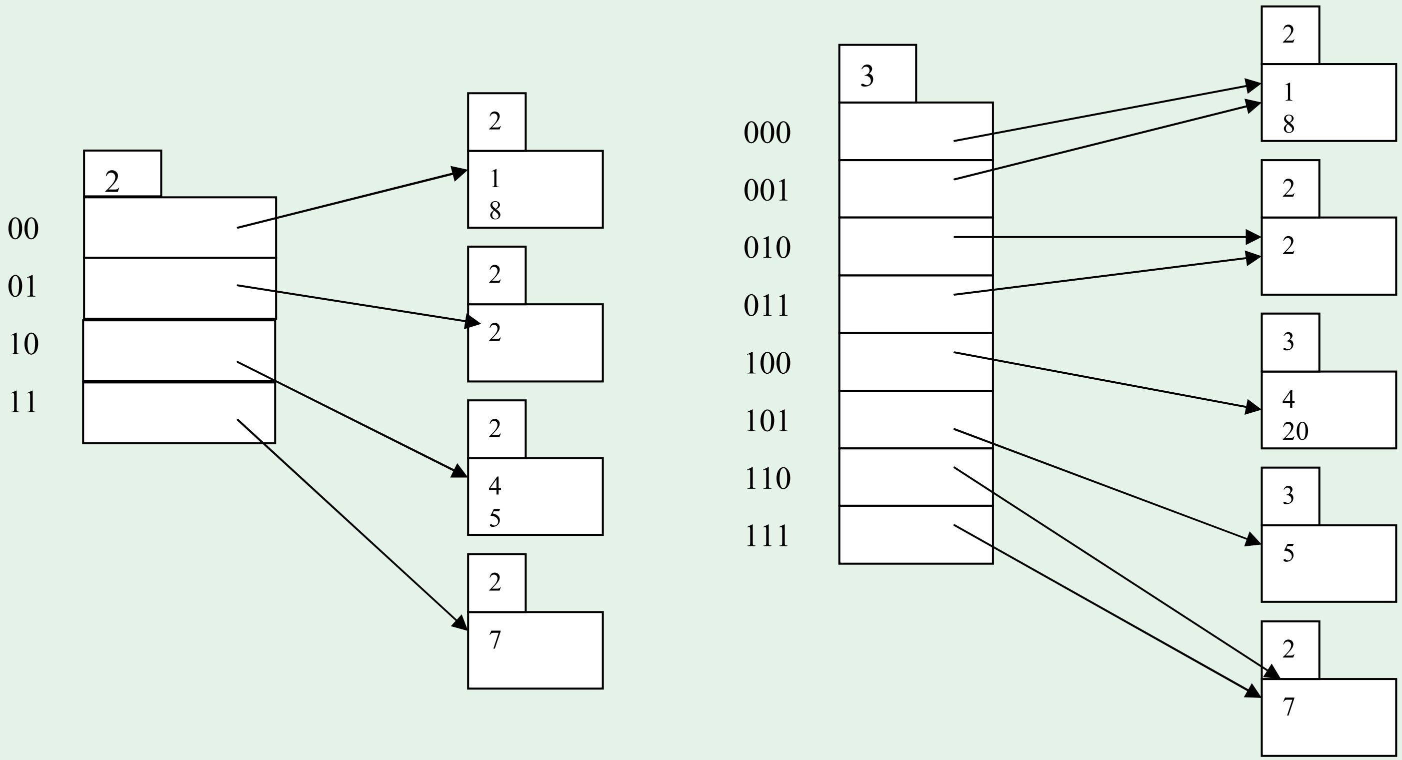
- Example 5: Suppose the hash function is ***h(x) = x mod 8*** and each bucket can hold at most two records. Show the extendable hash structure after inserting 1, 4, 5, 7, 8, 2, 20.

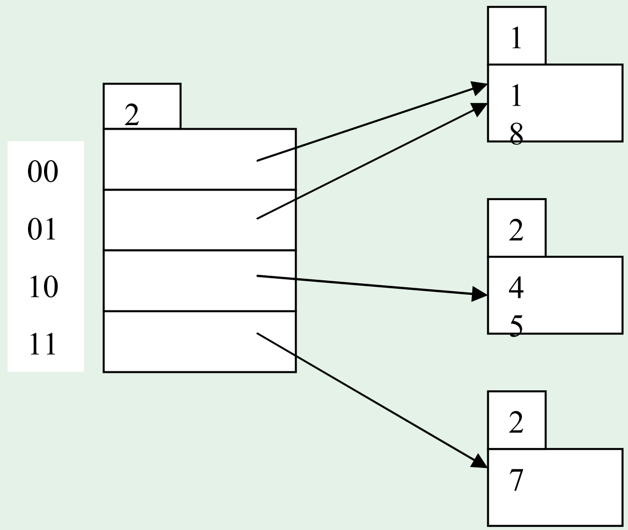
1	4	5	7	8	2	20
001	100	101	111	000	010	100



inserting 1, 4, 5, 7, 8, 2, 20

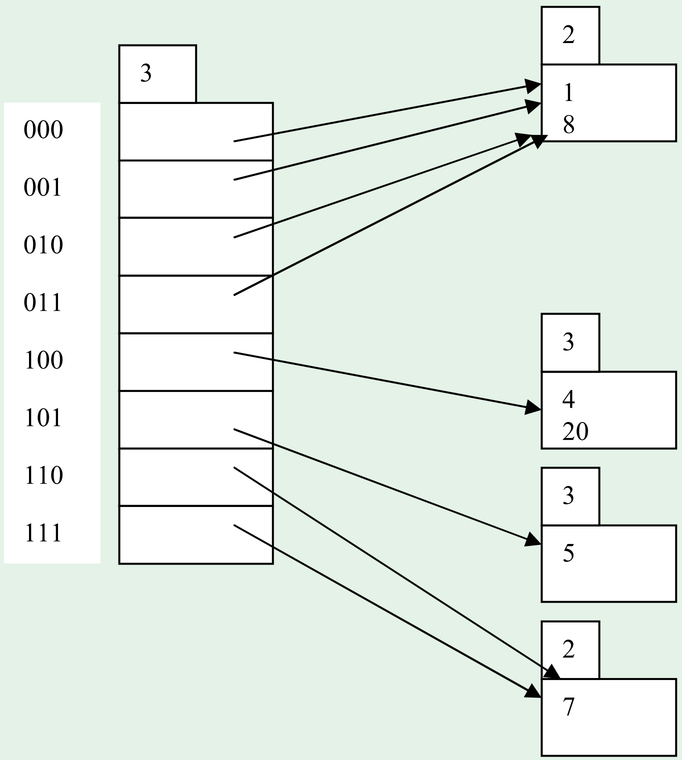
1	4	5	7	8	2	20
001	100	101	111	000	010	100





Suppose the hash function h(x) =x mod 8,
each bucket can hold at most 2 records.

Show the structure after inserting “20”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