NAME SURNAME, ID:

Duration: 70 minutes

Exam:
- This is a closed book, closed notes exam. The use of any reference material is strictly forbidden.

About the exam questions:
- The points assigned for each question are shown in parentheses next to the question number.

This exam consists of 6 pages including this page. Check that you have them all.

GOOD LUCK!

Question 1

Question 2

Question 3

Question 4

Total:
Suppose that you need to write a program to generate a single permutation of the first $N$ positive integers. For example if $N$ is 6, [3,1,2,5,4,6] is a legal permutation, i.e., each integer less than or equal to six should appear exactly once in this sequence. Assume you have a random number generator, \texttt{randomInteger(i,j)}, that can generate integers between $i$ and $j$. Analyze the average and worst case running times of the following algorithms using Big-Oh, i.e., $O(\ldots)$, notation.

(a) 10pts
Fill an array $a$ from $a[0]$ to $a[N-1]$ as follows: To fill $a[i]$, generate random numbers between 1 and $N$, until you get one number that is not in $a[0]$, $a[1]$, ..... $a[i-1]$
(b) 10pts
Same as the algorithm in part (a), but keep an extra array called the used array. When a random number, \( k \), is first put in the array, set \( \text{used}[k] = \text{true} \). This means that when filling \( a[i] \) with a random number, you can test in one step whether that number is used, instead of looking at all \( i-1 \) numbers before \( a[i] \).

(c) 10pts
Fill the entire array initially using \( a[i]=i+1 \). Then in a for loop, swap each entry with a random entry.

\[
\text{for } (i=0; i<N; i++)
\quad \text{swap} \ (a[i], a[ \text{randomInteger}(0, i) ]); 
\]
Write a recursive function, \texttt{count(a,x)} to count how many times a given integer, \texttt{x}, exists in an array, \texttt{a}, of integers. A non-recursive implementation uses a variable to store the temporary “count” as you go over the elements of the array. Your recursive implementation \textbf{must not} use any additional variable to store the result.

Example usage:

```c
int a[10] = {4,3,0,3,4,2,1,4,3,2};

printf("%d", count(a,4) );
```

should print “3” on the screen.
(a) 15pts
Given an example integer array of 10 elements in which quicksort performs in its worst case performance, \(O(n^2)\). Assume that we always use the middle element as pivot during the partition operation. You do not need to show the operations. But briefly explain why it will be \(O(n^2)\).

(b) 15pts
Describe an algorithm to sort an array of \(M+N\) integers in which the first \(M\) integers in the array are already sorted and the second part which contains \(N\) integers is not sorted. You may call existing sorting algorithms in your algorithm. What is the time complexity of your algorithm?
Give an example of a “class template” and an example of a “function template”. You do not need to provide any implementation.