

MIDDLE EAST TECHNICAL UNIVERSITY
Department of Computer Engineering

CENG567- Design and Analysis of Algorithms

Assignment 1

Due Date: March 23, 2006

Q1. (10 points) Show that for any real constants a and b , where $b > 0$,

$$(n+a)^b = \Theta(n^b)$$

Q2. (20 points) Solve the following recurrences using characteristic equation.

Assume that $T(n)$ is constant for $n \leq 2$

a) $T(n) = T(\sqrt{n}) + 1$

b) $T(n) = T(9n/10) + n$

Q3. (20 points) Suppose that instead of contracting a table by halving its size when its load factor drops below $1/4$, we contract it by multiplying its size by $2/3$ when its load factor drops below $1/3$. Using the potential function

$$\Phi(T) = |2 \cdot \text{num}[T] - \text{size}[T]|,$$

show that the amortized cost of a TABLE-DELETE that uses this strategy is bounded above by a constant.

Q4. (50 points) Find the lower bound of Convex Hull problem. Problem definition is: given a set S of N points in a plane, construct its convex hull; that is, the complete description of the boundary Convex Hull (S). Hint: you may use reduction method and the sorting problem to find the lower bound of convex hull problem. Your solution should be clear and include a simple example. If you use some sources, you must give the references.